DOCUMENT RESUME

ED 348 207 SE 049 639

AUTHOR	Mohyla, Jury, Ed.
TITLE	The Role of Technology: Theme Group 3. Proceedings of the International Congress on Mathematical Education (5th, Adelaide, South Australia, August 24-30, 1984).
INSTITUTION	Nottingham Univ. (England).
REPORT NO	ISBN-0-86803-205-0
PUB DATE	86
NOTE	210p.; A product of the Shell Centre for Mathematical Education. Printed at the South Australian College of Advanced Education, Adelaide, South Australia. Charts and drawing may not reproduce well.
AVAILABLE FROM	Shell Centre for Mathematical Education, University of Nottingham, Nottingham NG7 2RD, England.
PUB TYPE	Collected Works - Conference Proceedings (021)
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	Audiovisual Aids; *Calculators; College Students; Curriculum Development; Elementary Secondary Education; Foreign Countries; *International Educational Exchange; *Mathematics Education; *Mathematics Instruction; *Microcomputers; *Technological Advancement

ABSTRACT

This conference report bears witness to the centrality and growing significance of the debate on the impact of technology on mathematics education. It presents an overview of the conference's Technology Theme group's work, featuring summaries of the reports of 17 constituent working groups under 6 major headings: (1) "Challenges to the Curriculum"; (2) "Algorithms and Programming"; (3) "Television Video and Film"; (4) "Classroom Dynamics"; (5) "Teacher Education"; and (6) "Miscellaneous." The reports include international reviews, reports of the discussions, research findings, national views, personal views, and classroom explorations. The range and nature of the reports are varied and reflect the state-of-the-art in 1984. Thus, a spectrum of experiences ranges from the use of calculators through microcomputers to the use of powerful systems used to create cognitive models. However, a great difficulty in implementing curriculum change is noted; research and development projects must be linked to the varied social levers that might aid dissemination of tested practice. (MNS)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

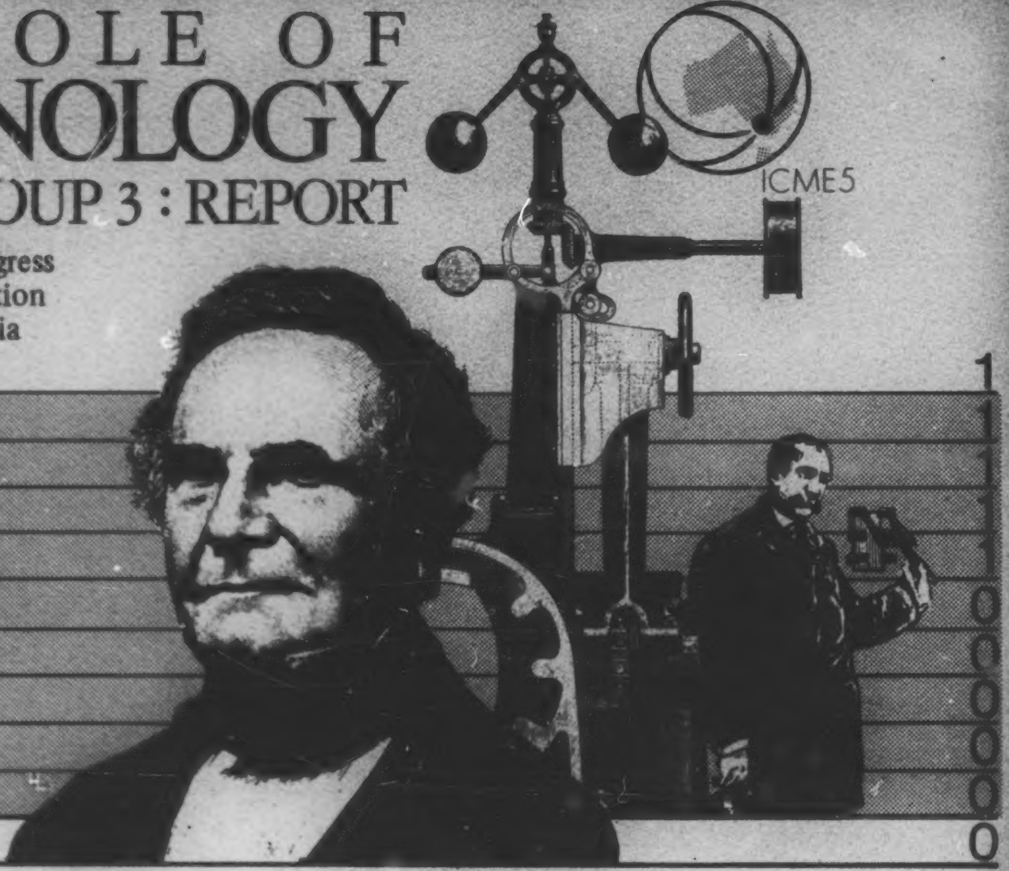
THE ROLE OF TECHNOLOGY

THEME GROUP 3 : REPORT

Fifth International Congress
on Mathematical Education
Adelaide, South Australia
August 24 to 30, 1984

Edited by Jury Mohyla

ED348207



U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☒ This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

● Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL IN MICROFICHE ONLY HAS BEEN GRANTED BY
R. E. Fraser
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

SE 049 639

BEST COPY AVAILABLE

ICME 5 CONGRESS

Adelaide, 24 August to 30 August 1984

THE ROLE OF TECHNOLOGY: THEME GROUP 3

Chief Organisers

Rosemary Fraser (UK)
Hartwig Meissner (West Germany)
Tony Ralston (USA)
David Roseveare (UK)

Australian Co-ordinator

Jury Mohyla

Editor

Jury Mohyla

©ICME 5
ISBN 0-86803-205-0
Printed at the South Australian College of Advanced Education,
Adelaide, South Australia

TABLE OF CONTENTS

Introduction and Overview v

Challenges to the Curriculum 1

Algorithms and Programming 61

Television Video and Film 89

Classroom Dynamics 93

Teacher Education 161

Miscellaneous 185

Introduction and Overview

Jury Mohyla, Peter Brinkworth (Australia)
Rosemary Fraser (UK)

The Fifth International Congress on Mathematical Education (ICME 5) was held in Adelaide, South Australia, 24-30 August, 1984, and incorporated the Tenth Biennial Conference of the Australian Association of Mathematics Teachers. The Congress was organised on behalf of the International Commission on Mathematical Instruction (ICMI) through the Australian Subcommission of ICMI and the Australian Academy of Science. Previous Congresses were held in Lyons, France (1969), Exeter, UK (1972), Karlsruhe, West Germany (1976) and Berkeley USA (1980). ICME 6 is planned to be held in Budapest, Hungary in 1988.

The 1984 conferees included 198 "accompanying persons", and of the 68 countries represented, major groups were 814 from Australia, 251 from the USA and 173 from Japan.

The three major plenary addresses were given by Ubiratan D'Ambrosio (Brazil) on Socio-Cultural Bases for Mathematics Education , Jeremy Kilkpatrick (USA) on Reflection and Recursion and Renfry Potts (Australia) on Discrete Mathematics, while the Presidential Address was delivered by Jean-Pierre Kahane (France) on Measures and Dimensions. The text of these addresses, together with summaries and reports of other plenary sessions, presentations and various theme, action, and study groups, are contained in the Proceedings of the Congress. The very full program also included lectures, film screenings, meetings, project displays and presentations, short communications and poster sessions, although many significant interchanges took place at times set aside for informal social activities, such as Coffee Breaks and Happy Hours.

One of the most significant features of the Congress was the venue which provided the first opportunity for an ICME to be held in the Southern Hemisphere. Another was the decision of the organisers to promote small group dialogues through Action Groups and Theme Groups. While the Action Groups were organised by levels, early childhood, elementary, junior, secondary, senior secondary, tertiary, teacher education and adult education, the Theme Groups were organised by topic: maths for all, teacher development, technology, theory-research-practice, curriculum, applications and problem-solving. Participants elected to attend meetings of smaller groups within their chosen Action and Theme Groups, so that current knowledge could be both exchanged and synthesised. In most cases, debate and discussion were provided by short initial presentations by participants.

Of the many themes highlighted at the Congress, the technology theme was perceived to be the most central and visible, not only through theme group discussions but also through invited presentations, displays, exhibitions and the use of technology itself throughout the Congress. The publication of this report bears witness to the centrality and growing significance of the debate on the impact of technology on mathematics education. It aims to present an overview of the ICME 5 Technology Theme Group of the constituent working groups. A brief summary of the Theme Group's work is also contained in the Congress Proceedings.

Under the technology theme at ICME 5 seventeen working groups were offered under six major headings

- Challenges to the Curriculum
- Algorithms and Programming
- Television Video and Film
- Classroom Dynamics
- Teacher Education
- Miscellaneous

Brief summaries indicating the nature of the report included in this book for each working group are given below.

The range and nature of the reports are varied and reflect the 'State of the Art' of the particular topics explored. Whereas many school based projects concerning the use of calculators are reprinted, giving the possibility of calling for 'action' based on experience the report dealing with symbolic mathematical systems is of necessity more speculative, with experience so far mainly in the province of teaching and research level.

Thus we have a spectrum of experiences to exchange ranging from the use of calculators through microcomputers to the use of powerful systems used to create cognitive models.

However, one observation worth making is that even where we have evidence based on research findings together with consensus amongst the community of math educators we still have great difficulty in implementing curriculum change. With the pace of technological change it becomes imperative that we promote parallel research and development projects linked if possible to the varied social levers that might aid dissemination of tested practice.

The reports include international reviews, reports of the ICME discussions, research findings, national views, personal views, classroom explorations at both tertiary and secondary levels. One objective of the publication is to share ideas between researchers, educators and teachers at both tertiary institutes and schools.

Summaries of the Working Groups

1 Challenges to the Curriculum

The Calculator Report

1.1 Calculators for Developing Countries

1.2 Calculators for Developed Countries

1.1/1.2

This report presents an international survey of the use of calculators in mathematics education. In summary the findings are

(a) There has been much less research activity concerning calculators since ICME 4 (Berkeley) than there was between ICME 3(Karlsruhe) and ICME 4.

(b) Research investigations since 1980 have not produced dramatic new insights or results, they mainly confirm the findings of previous investigations.

(c) The overwhelming majority (greater than 95%) of investigations indicate that the use of calculators does not harm mathematics achievement in terms of the traditional curriculum and traditional tests.

(d) There are many countries with the availability rate of calculators to children of more than 80% where the school curricula ignore the existence of calculators or where the calculator use in mathematics education is not allowed for many grades.

1.3 Technology and Secondary School Maths Curriculum

The effort to infuse mathematics teaching with appropriate computer use faces several barriers. At the present time, in-school computer facilities in many countries are inadequate to meet the needs of an appropriate curriculum. This is further complicated by lack of well prepared teachers capable of implementing the new technology strategies. These are some of the major points discussed by this group as well as the need to develop effective computer software, explore alternative curriculum materials and focus on in-service education of teachers.

1.4 Symbolic Mathematical Systems and their effect on the Curriculum

Twelve papers concerned with the creation of and experiences with symbolic mathematical systems were proposed for the consideration of this working group. Contributions were received from Austria, France, USA, Australia, Switzerland, Germany, Finland, Italy, Netherlands and England. As these papers are published in full in SIGSAM (Special Interest Group on Symbolic and Algebraic Manipulation) publications, only their abstracts are printed in this book.

The ICME group paid particular attention to the question of how current symbolic mathematical systems should be improved in order to meet the particular requirements for being valuable tools for mathematical education.

It is true to say that the current symbolic systems have been developed by the computer scientist, research is now needed to develop an interface that will enable them to become accessible to teachers and students. Currently the systems are of most use to the research mathematician and scientist.

2 Algorithms and Programming

2.1 Programming Languages for Mathematics Education

A short summary of the group discussion is given. The major points mentioned are

(a) From the perspective of the mathematics educator computer languages should be seen essentially as tools rather than as objects in their own right.
(b) As has been historically true with other educational innovations, there would seem to be considerable difficulties associated with the transfer of this technology to the developing world.
(c) Two principal rules for programming were identified:

    * As an arena for student-directed exploration
    * As a medium for the development of teacher or curriculum support materials.

2.2 Algorithmic Approaches to Mathematics Education

A short summary of the group's discussion is given. The major headings are

(a) General issues in the algorithmic approach.
(b) The language of algorithms in the teaching of mathematics.
(c) Efficiency of an algorithm.
(d) Problem Solving with and without algorithms.
(e) Known Experiments in development algorithmic thinking.

2.3 Technology and Cognitive Development

The group reports on information disseminated, discussions and recommendations elaborated during the four working sessions at ICME 5. The report focuses on three questions:

(a) The use of technology to further the cognitive development of learners of mathematics.
(b) The use of technology to achieve a better understanding of the processes involved in children's development of cognitive structures relevant to mathematics.
(c) Possible disadvantages connected with the use of technology.

A comprehensive set of references is included.

3 Television Video and Film

3.1/3.2 Film and Video in Mathematics Education
Film and Video about Mathematics Education.

This group presents a short summary of their discussions which was grouped under nine main headings: Hooking, Signposting, Scripting, Coherence, Syncopating with the viewer, Active learning, Attuning, Range of Viewer ability, Ending.

The 30 participants aim to set up an international newsletter.

4 Classroom Dynamics

4.1 Feedback to the learner

This group considered a possible classification of the types of feedback it is possible to offer a learner, both with and without technological resources. A classification of feedback was drawn up by the group.

4.2 Roles of the Microcomputer in the Mathematics Classroom

This group was somewhat different in nature other groups; they held four workshop sessions. Six subgroups selected one computer program, prepared and presented 'lessons' to the whole group. This focused discussion on the dynamics of the classroom and highlights the need to develop curriculum applications rather than isolated computer programs.

Six major classroom rules were identified and the sharing of these rules between the teacher, the pupils and the microcomputer are analysed for each program explored. A selection of prepared papers is included.

4.3 Learning Environments

This group discussed different learning environments which make use of a microcomputer and considered how these would effect students' learning.

The report records differences of opinion on a number of fundamental issues, agreement on a number of points and an optimism tempered by an awareness of many practical obstacles. It might take a whole generation before the full benefits of technology reach the classroom.

The contributed papers published deal mainly with experiences of a 'Logo' environment!

5 Teacher Education

5.1 Computer and In-Service

Report is published in the ICME publication Professional Life of the Teacher (Theme group 2)

5.2 Computers and Pre-Service Teacher Education

This group reported under four major headings:

(a) The State of the Art in Different Countries.
(b) Teacher Education in Computers in an Open System.
(c) Effects on Mathematical Education.
(d) Problems in Schools and Society

Countries presenting views were USA, France, Germany, China and New Zealand.

6 Miscellaneous

6.1 How to Start with Computers

This group reports under the headings:

(a) Hardware Factors
(b) Didactical Aspects of Beginning with Computers in Mathematics.
(c) Organisational Aspects
(d) Software to Serve Students.

The report included discussion and also reports classroom case studies.

6.2 Technology for Education of the Handicapped

This group did not attract particpants and was therefore cancelled.

6.3 Technology as an Aid to the Teaching of Problem Solving

Report is published in the ICME publication Problem Solving (Theme group 7).

CHALLENGES TO THE CURRICULUM

CALCULATOR REPORT

1.1 Calculators for developing Countries

1.2 Calculators for developed Countries

Group Members:
Bengt Anderberg,
Karl Greger. Sweden
Donna Berlin,
Arthur White. (USA)
Allen Edwards.
Papua New Guinea
Russell Langham. England
Alwyn Olivier. South Africa
Pilar Vela. Mexico
Hans Brolin,
Kazu Aoki. Japan
Len Miller,
Lucilla Cannizzaro. Italy
Ed Jacobsen. UNESCO
Ilpo Laine. Finland
Hartwig Meissner. West Germany
James K. Taole. Lesotho
Chair/Reporter: Hartwig Meissner

1.1.1 **Introduction**
Hartwig Meissner, Westf. Wilhelms-Universitat Munster, FRG
1.1.2 **An International Survey**
1.1.2.1 Calculators in Italy, France and Belgium
Lucilla Cannizzaro, Universita di Roma, Italy
1.1.2.2 Calculators in Sweden
Hans Brolin, Uppsala Universitet, Sweden
1.1.2.3 Calculators in West Germany (FRG)
Hartwig Meissner, Westf. Wilhelms-Universitat Munster, FRG
1.1.2.4 Calculators in South Africa
Alwyn Olivier, University of Stellenbosch, South Africa
1.1.2.5 Calculators in Japan
Kazuyoshi Aoki, Chuo University Tokyo, Japan
1.1.2.6 Calculators in the USA
Donna F Berlin and Arthur L White,
The Ohio State University Columbus, Ohio
1.1.2.7 Calculators in Other Developed Countries
Hartwig Meissner, Westf. Wilhelms-Universitat Munster, FRG
1.1.2.8 Calculators in Developing Countries?
Hartwig Meissner, Westf. Wilhelms-Universitat Munster, FRG
1.1.3 **Some Calculator Projects**
1.1.3.1 The Numeracy Project of Papua New Guinea (PNG)
Allen Edwards, Administrative College of PNG, Boroko-PNG
1.1.3.2 Calculators in the Math Lab
Arthur L White and Donna F Berlin, OSU Columbus, USA
Pilar Vela, Colegio Americano, Puebla Mexico

1.1.3.3 Project "Keystrokes"
Arthur L White and Richard J Shumway,
The Ohio State University, Ohio USA
1.1.3.4 Calculators Count
Russell A Langham, Senior Mathematics Adviser, Durham G.B.
1.1.3.5 Project TIM
Hartwig Meissner, Westf. Wilhelms-Universitat Munster, FRG
1.1.3.6 ARK
1.1.3.7 GDR Project
1.1.4 **Summary**
1.1.4.1 Research Findings
1.1.4.2 Proposal for Curriculum Change
1.1.4.3 Results ICME 5
1.1.5 **References**

CALCULATOR REPORT

1.1 Calculators for developing Countries

1.2 Calculators for developed Countries

Group Members:

Bengt Anderberg,
Karl Greger. Sweden
Donna Berlin,
Arthur White. (USA)
Allen Edwards.
Papua New Guinea
Russell Langham. England
Alwyn Olivier. South Africa
Pilar Vela. Mexico

Hans Brolin,
Kazu Aoki. Japan
Len Miller,
Lucilla Cannizzaro. Italy
Ed Jacobsen. UNESCO
Ilpo Laine. Finland
Hartwig Meissner. West Germany
James K. Taole. Lesotho

Chair/Reporter: Hartwig Meissner

1.1.1 Introduction

Hartwig Meissner, Westf. Wilhelms-Universitat Munster, FRG

This report is a team work of more that 15 specialists from 10 countries. We describe the present situation, based on the knowledge of the authors, on reports received from other specialists, on preceeding documents, and on the output of the ICME 5 working sessions
Simple "Calculators" are no longer a central theme at congresses or conferences on mathematics education. "Computers" and "Problem Solving" are the hits at the moment. The preparation for ICME 5 included two working groups on calculators, one for developed countries, one for the needs of developing countries. Only 16 conference participants joined these two groups which therefore were merged into one group.

During the first two sessions in Adelaide the experts from different countries gave state-of-the-art reviews. The national reports were complemented by survey reports on calculator activities and research results in Europe and in North America. Since most commonly, calculator use is restricted to use as a calculation device, the working group collected examples for a more creative use by giving examples from different calculator projects. There was a general agreement that the majority of teachers and mathematicians still ignores the educational power of calculators. They even persist with the idea that calculators will harm mathematical achievement though there is no eavidence for that statement according to almost all the research results.

This report therefore will inform. In part 1.1.2. we give an international survey about the use of calculators in mathematics education if different countries. A speical paragraph is devoted to the very different problems in developing countries. In part 1.1.3. we describe calculator projects from seven countries in more detail. Some of the projects were organized nationwide. Their design, their evaluation, and their findings are impressive.

In part 1.1.4. of report we first summarize researach findings, mainly from investigations conducted in the USA. We then reprint a paper given at the AERA conference in 1979. This paper discusses the needs of a curricular change and it is still of immediate interest. The last paragraph summarizes results from ICME 5 and part 1.1.5. lists the references.

1.1.2 An International Survey

1.1.2.1 Calculators in Italy, France and Belgium

Lucilla Cannizzaro, Universita di Roma, Italy

In Italy, France and Belgium no specific research on the use of calculators in elementary or junior secondary grades specifically has been devoted to collect informations on how pupils react to the calculator, or to measure skills and abilities in mental arithmetic of paper and pencil algorithms while using the calculator. General descriptions on calculators' advantages are described in references APM82, APM83, UMI82. Many experimental work has been done in order to introduce heuristic procedures of learning/teaching as well as changes in the syllabuses, or in parts of them.

The RICME project RIC79 for elementary schools does not use calculators explicitly but introduces and largely uses an algorithmic and heuristic way to implement mathematical concepts. Also because of this project the new Italian programs for elementary schools (ages 6-10), which will be implemented during 1986, suggest (not impose) the use of calculators.

The Genova Projects for junior secondary schools (se GPR83, B/G84) took in account calculators as an aid to develop specific curriculum sections, specially non mathematical ones. The '79 new programs MIN79 explicitly ask for the use of calculators, algorithmic approaches and flow diagram languages.

In all three countries there are smooth revisions of all the curricula. There are projects devoted to implement calculator materials for developing arithmetic skills in a specific way, but, at the present, they are rather isolated (see VAN78, H/V82, IRR83). There is a general agreement in underlining the following main advantages in using calculators or algorithmic approaches (see also TOR82, NOE83):

a) Calculators can help less able children to gain in computation abilities, depending on the pupils' ages (also concerning motivation to work!).
b) Calculators help to train basic facts (addition, multiplication, order of magnitude, rounding) and to explore mathematical concepts at earlier ages (negative numbers, deacimals, fractions, irrational numbers, or properties of functions and relations between and within the domain and the range).
c) Calculators realize significant gains in mental arithmetic and the consciousness that mental arithmetic still is necessary. They imporve computational skills by working on many examples; they allow the introduction of new topics which had a too big

computation volume; they help to use more realistic and interesting numbers.

d) Calculators help to develop a number sense by working with the 4 operations in calculator games; they put less emphasis on long algorithms (for instance operations with fractions); they stress problem solving abilities (checking conjectures immediately and avoiding misunderstandings, asking not 'how to multiply', but 'when' or 'what to multiply').

e) Calculators help to develop a sense of function and realize significant gains with students by using function ideas via guess and test procedures, they put greater emphasis on relations between functions (for instance inverse functions), or, in a more general way, on sequences of operators.

f) Calculators develop algebraic languages and skills generating examples, generalizing and looking for equations, checking for operation properites and syntax rules in a more spontaneous way.

g) Taking in account classroom procedures and dynamics for developing mathematical skills, the use of a calculator also can be observed as a tutor (for instance as a feed back machine, covering the display, to check results or allowing to press only a few keys).

1.1.2.2 Calculators in Sweden

Hans Brolin, Uppsala Universitet, Sweden

When the pocket calculator was introduced into Swedish Classrooms, mathematics teachers were faced with many new problems. The National Board of Education in 1976 had the foresight to appoint the ARK group whose task was to analyse the effects of the pocket calculator on mathematics teaching throughout the school system. Till now the ARK project has

a) compiled and tested test series designed to establish whether pupils' concept of numbers and ability to solve problems had been affected - particularly in the lower grades,

b) analysed and tested the possibility of improving pedagogical methods in certain sections of the curriculum by employing the pocket calculator,

c) tested new, high priority changes in the curriculum made possible by the pocket calculator.

The ARK project has served as an umbrella project for ten subprojects covering all school grades. For more details, conclusions, and recommendations see part III.F.

1.1.2.3 Calculators in West Germany (FRG)

Hartwig Meissner, Westf. Wilhelms-Universitat Munster, FRG

In the Federal Republic of Germany calculators are mainly used beginning at grade 7 (age 13-14). In some states they are allowed (as teaching "aids") even in elementary grades, in others they are forbidden till grade 9. Traditional calculation devices like slide rules or tables have been disappeared from schools.

Till now there has not been a bigger change of curricula according to the presence of calculators. Calculators mainly are used for computations only. Very few attempts are made to integrate the educational power hidden in calculators into the classroom work (discovery processes, guess-and-test procedures, simulation of functions, estimation and verification, ...). There are experiments (see AMV78, LAN79) to use calculators for training mental arithmetic. Other experiments indicate, that calculators can be used for developing an intuitive number sense, a "percentage feeling", or an intuitive "function sense" (see MEI78, MEI85). Calculators are allowed in tests whenever calculation is not a goal of the test. There are investigations (see W/W80, WYN84) which show that the use of calculators does not harm computation abilities and skills.

1.1.2.4 Calculators in South Africa

Alwyn Olivier, University of Stellenbosch, South Africa

Although calculators have been increasingly available in South Africa for many years, their introduction into the mathematics classroom was not sericously considered until recently.

At the senior secondary school level the Cape Education Department was engaged in two experiments (1979 and 1981) in which several schools used calculators in regular mathematics instruction as well as in examinations. Research indicated positive results on computational skills, attitudes and concept development.

An important barrier to the widespread use of calculators in classrooms was overcome when the joint matriculation board, the watchdog for university entrance requirements, permitted the use of calculators in their examinations in 1983. Other examining bodies soon followed the trend. The use of calculators furthermore is facilitated by the introduction of a new centralised curriculum in 1985 which includes the use of calculators. Most education departments provided inservice training to familiarise teachers with calculators. However, apart from dropping logarithm tables, the curriculum has not taken into account the potential of the calculator to introduce new topics or to approach traditional topics in new ways.

At the primary education level the use of calculators in schools is not an issue: they are simply being ignored. The reasons are the classical ones: fear that achievement will be harmed and a beleif that basic skills should first be mastered before the use of calculators is allowed. This is symptomatic of an ignorance of the potential of the calculator as an instructional aid and of the vast body of research evidence supporting the use of calculators at all levels. This should change as teachers learn about and personally explore more significant roles for calculators.

1.1.2.5 Calculators in Japan

Kazuyoshi Aoki, Chuo University Tokyo, Japan

Whoever visits Japan will be surprised about the popularity of using

1.1.2.5 Calculators in Japan

Kazuyoshi Aoki, Chuo University Tokyo, Japan

Whoever visits Japan will be surprised about the popularity of using calculators. Business workers always carry them in their pockets. Even stall-keepers make use of them. Children play games of simulation type, alone or together, with displayed "game-watches". One may wonder why Japanese schools keep away from calculators in their curriculum. There are some reasons.

a) People favour mental training in arithmetic. Children should master 9x9 recitation and manual calculation in their early grades (1-3). Calculators are therefore regarded as unwelcomed facilities within the lesson. The SOROBAN (Japanese abacus) still remains in the curriculum, for it's use is believed to develop mental calculation abilities.
b) Learning the algorithms of fractions is a central theme in the middle grades (4-6) and it is essential that calculators are not able to give answers in fraction form.
c) In the higher grades there are fewer chances for the apparatus to play an active part in the mathematical lessons, because letters start to take the place of numbers.

Thus the only possible turn for a calculator to show its abilities may be in processing data for statistical studies. Function keys to get the mean and the standard deviation are essential then.

Meanwhile programmable calculators are coming into the market. Japanese makers invent new models one after another, adding more convenient devices to hand-sized bodies. Using these models one can perform mathematical exercises in an ordinary classroom.

The reporter has developed a course of programs for exercises on elementary probability and statistics making the most of arithmetic random numbers, which are applied to simulate various phenomena e.g. dice tossing, frequency of accidents, car flows, random walks and error estimation. Each such program needs 10 to 15 short orders, so that students can master the first step of computer operation besides the subject itself. To one's joy, the longer one will run the program, the better results can be expected, thanks to the law of large numbers.

This method will be popularized in high schools and colleges, as programmable calculators may soon become within students' availability.

1.1.2.6 Calculators in the USA

Donna F Berlin and Arthur L White,
The Ohio State University Columbus, Ohio

The study of the use of calculators in the USA has given way to the popularity of the microcomputer for classroom use. The research into calculator use has generally alleviated concern about the potential negative effects on children's development of mathematical concepts. Consequently, calculators have taken their place among a

variety of educational tools in the elementary school mathematics curriculum. The decrease in cost and general availibility of calculators in the USA along with the development of resource materials to guide teacher use of calculators in elementary school mathematics has helped contribute to the general acceptance and integration of calculators in the school curriculum.
Some elementary mathematics programs are using calculators in a regular, systematic way; others assign calculators to a supplementary role for remedial or enrichment activities. Some schools have included calculator activities as a part of the elementary school mathematics laboratory program and/or resource center. The calculators are often employed to teach concepts of numeration, place value, number sequence/patterns, integers, decimals, geometry, squares, square roots, probability, estimation, measurement, and as a computational tool in problem-solving experiences.

1.1.2.7 Calculators in Other Developed Countries

Hartwig Meissner, Westf. Wilhelms-Universitat Munster, FRG

Calculators are used (in daily life or business) wherever they are available. In many countries the prices for calculators are approximately the same as for books, the distribution of calculators almost is uncountable. Therefore in many countries we also can find calculator experiments in schools. For example the number of calculator projects in the USA is almost as big as the number of school districts. "Calculator Information Centers", specially the one in Columbus, Ohio 43212 USA (Director Marilyn N Suydam, 1200 Chambers Road) collect and disseminate detailed information.

The Shell Centre for Mathematical Education in Nottingham acts - besides other project groups - as a research and development center for Great Britain. Members of that center carried out one of the first studies in a primary school. Meanwhile there are a lot of materials availible for schools, for inservice courses and for distance learning (for example see MER84, T/884, CPS82).

There are some reports, which give summaries upon different countries and state-of-the-art reviews. We mention SUY79 with reports on Australia, Austria, Belgium, Brazil, Canada, Federal Republic of Germany, Hong Kong, Ireland, Israel, Japan, New Zealand, Sweden, Switzerland, Thailand, United Kingom, USA. An overview upon calculator projects mainly in eastern european countries is given in WAL84 (German Democratic Republic, Russia, Poland, Hungary, Czechoslovakia, but also Austria and Federal Republic of Germany). Another profound analysis about calculators in mathematics education was put together by Peter Knopf (see KNO80) with summaries from the USA, Switzerland, Federal Republic of Germany, Great Britain, Denmark and Sweden.

Summarizing we can state that calculators are well accepted in many developed countries. Calculators mainly are used as computation devices. There are research projects to discover the educational potential of calculators but in most countries the school boards are very reluctant in changing the curricula.

1.1.2.8 Calculators in Developing Countries?

Hartwig Meissner, Westf. Wilhelms-Universitat Munster, FRG

a) Real World Situations

People live, cook, and sleep in small one-room-huts, sometimes together with dogs, cats or other domestic animals. They get their water from a spring or a well and heat by burning wood or other materials. The huts are built (of course without machines) by clay or wood or bamboo. Is this the world to use technology in mathematics education?

Travelling through these countries we met "strange" situations. Buying 10 eggs (7 currency units each) in a "super market" the young lady added on a cash machine "7+7+7+7+.... (counting with her fingers)...+7+7 = 70" (Central America). Changing money in a bank I had to join 3 different counters. At counter A and counter B the employees independently had to compute the exchange rates with a calculating machine, at counter C the employee had to compare the two different printouts (Africa). Passing a small village of bamboo huts (without electricity) we saw several huts with TVs (South East Asia). Is this the world to introduce calculators into mathematics education?

b) Technology and Environment

Technology is a question of money and mentality. People who face hunger and poverty never will buy a calculator. But people who have a TV or motorbike or a car of course also will have a calculator if there is a need for it. Talking about calculators for developing countries we must specify the situations we mean.

We cannot think of pupils in the bush sitting on the ground in the shadow under a large tree with calculators in their hand and listening to their teacher. We also do not mean pupils in well equipped school buildings in the capital city. We think more of rural central schools where the pupils live during the week or even during the school year. We think of overcrowded schools in poor suburban areas. And we think of children who only have a few years to learn "mathematics ".

c) Learning Conditions

Can the calculator facilitate or accelerate this process of learning mathematics? And what kind of mathematics? Experiences from developed countries show, that there is almost no schooling necessary to use a calculator for the four basic functions. Children have sufficient "theoretical" number sense (base system, arithmetical operations) and they "understand" what 1427 x 298 means. Is that also true for developing countries with a more "practical" number sense, mainly based on personal experiences? What do children in these countries understand when reading "1427 x 298"?

We can use the calculator in two modes. Using the syntactical mode we work on a sequence of digits (digital data). According to the underlying algorithms we press button after button, often forgetting the preceding ones. We work on a problem step by step without reflecting the meaning of it. (The same behaviour

BEST COPY AVAILABLE

can be observed while doing paper-and-pencil algorithms).In the semantical mode we work on relations between input and output (analogous data). We use guess-and-test procedures to emphasize the order of magnitude of input and output. The "wrong" output gives hints for a better next input. Thus we get a feeling how to vary input intervals Δ_1 according to ouput intervals Δ_2. We get" Δ -experiences" (see MEI83).

Most "practical" calculator use is syntactical. Problems may arise if there is no appropriate number sense (see examples "exchange rates" or "buying eggs") or if there is not enough "theoretical" knowledge about mathematical back ground concepts.

d) The Task

Introducing calculators in developing countries is not the main question. The more important problem is to introduce an effective use of calculator education at all and to all, i.e. to all children. How much computation abilities/skills do they need? What is the number sense, the function sense they should get? Which mathematical concepts are realistic? What can be the role of the calculator for this kind of curriculum? Can the calculator help to reduce the time of schooling?

There are other problems to face: Mentalities, financial and maintenance problems, teacher preservice and inservice, student books and work sheets,. language problems,....

To introduce calculators in mathematics education in developing countries is a much more complicated task than in developed countries. So far as I know this challenge has not been faced on a larger scale till now.

1.1.3. Some Calculator Projects

1.1.3.1 The Numeracy Project of Papua New Guinea (PNG)

Allen Edwards, Administrative College of PNG, Boroko-PNG

The Numeracy Project of Papua New Guinea was set up in 1980 by the Department of Commerce to help the many adults who wished to run trade stores but lacked basic number skills. Indeed, in Papua New Guinea, there is virtually no "ethno-numeracy" apart from 250 different systems of counting. Even the minority who have completed Primary education often lack all number skills beyond simple addition.

The most possible solution lay in the introduction of the calculator. Detailed teaching on its use however was essential and this necessitated the provision of a standard model. One was selected and the Project has since sold some 10 000 of these standard calculators at about half the usual retail price. During 1980-1982 considerable research was done (for example see EDW82) on the logistics of introducing calculators into villages and on the preparation of teaching materials.

In 1983 the Numeracy Project was transferred to the Administrative College of Papua New Guniea. Here the emphasis was on training Government Officers, who generally had secondary education. Their numeracy skills however were still exceedingly weak and it was clear that they needed both instruction in the use of the calculator and some means of developing their "number sense". It seemed obvious that this latter development would only come about through the use of mental arithmetic associated with real life problems. This led to the teaching of estimation skills. In practice this meant devising techniques which were reasonably accurate, easy to carry out mentally, and above all, "markable". Substantial development followed (EDW84) and the resultant series of "Calculate and Estimate" courses appear to be the first texts which fully integrate the use of the calculator, the teaching of estimation and the application of these skills to real-life problems. Many techniques of calculator use are quite sophisticated and if the instrument is viewed as a tool for obtaining results efficiently then a great deal of teaching is needed in association with specific applications. In the current courses for example, using only a basic calculator the following techniques are introduced: correction of keying errors, repeated addition using +, = = = ..., the memory, the % key for simple and compound interest, mark-up and profit % etc., finding a difference when the smaller number is already in the display and a quotient when the divisor is. The constant function is used frequently, particularly for finding proportional sets which are used in constructing graphs etc. Various algebraic formulae which would not be understood e.g. in the amortisation of loans, are converted into keying instructions which "mean" as little but which, unlike the algebraic formulae, can in fact be used in real life situations by ordinary people.

The logistics of teaching in a non-formal situation have been a major concern of the Numeracy Project. One course was designed for teachers to use in a classroom, with very detailed instructions in a Teachers Manual. The other courses have used a self-paced format but the inadequacy of this in itself was quickly appreciated. Trainers have therefore had to be trained to use such courses. Finally audio-cassettes were produced to relate specifically to workbooks and the use of these has proved very successful, but it is anticipated that true distance learning even using these tapes, will still require the setting and marking of mastery tests.
The Numeracy Project has been privilged to research and develop materials in what is probably the least numerate of any surviving culture. The use of the calculator has been fundamental in its response to these needs. But experience gained in a country where the needs of the non-numerate are paramount, may yet be helpful in other countries where the non-numerate form an often neglected minority.

The combination of mental arithmetic and estimation skills with the efficient use of the calculator appear to the Project Co-ordinator to be the twin foundations on which practically the whole of applied arithmetic could be re-built, not only in the developing but also in the more developed parts of the world.

1.1.3.2 Calculators in the Math Lab

Arthur L White and Donna F Berlin, OSU Columbus, USA Pilar Vela, Colegio Americano, Puebla Mexico

The mathematics laboratory program in the primary grades of the American School in Puebla, Mexico, was initiated for the purpose of providing an activity-orientated approach incorporating the use of calculators to supplement the mathematics instruction in the regular classroom. The ultimate goal of the program is to enable children to master basic abstract concepts to be used later in higher order, more complex concepts and processes. At its inception, the program was targeted for students experiencing difficulty in the regular mathematics program (i.e. a remedial program). As the mathematics laboratory got underway, it became evident that this approach using calculators and manipulative materials was very different from the activities that students were experiencing in the regular classrooms. Recognizing the potential benefits of the program for all students led to its extension to all primary level students. In addition, the use of the mathematics laboratory by all students created a positive image and removed the stigma associated with the laboratory's original use for remediaton only.

The prototype for the development of the mathematics laboratory in Puebla, Mexico was a program that was developed in 1979 in the Lincoln Heights Elementary School, Princeton City School District, Cincinnati, Ohio. The use of the calculator and manipulative materials (e.g. abaci, attribute blocks, base ten blocks, bundling sticks, chip trading kit, colored cubes, geoboards pattern blocks pegboards) have been integrated for the teaching of fundamental mathematics concepts, skills, and processes.

In both Cincinnati, Ohio and Puebla, Mexico the mathematics laboratory is a separate, self-contained room in the school building. Students are scheduled by class to come to the laboratory one period a week for 20 to 40 minutes. The laboratory is set up with three tables with chairs for 6 to 8 children at each table. Three different activities using the calculator and manipulative materials are prepared daily for each of the three tables. The activities and materials selected relate to a common mathematical concept. For example, activities related to the concept of place value may include base ten blocks at one table, chip trading at another table, and calculator counting activities at the third table. Each day groups of children are assigned to an activity and then rotated to different activities on subsequent days so that all children experience all the activities. The regular classroom teacher is present in the mathematics laboratory to assist with the activities. In addition, the classroom teacher is encouraged to make use of the calculators and manipulative materials in their daily mathematics lessons. Planning sessions are also provided so as to integrate the laboratory activities with the regular classroom mathematics instruction.

Evaluation of the prototype program in the Princeton City Schools was extensive during the 1980-1981 school year. The time spent by children on activities using the calculator and manipulative materials was correlated with the year's growth by item reflected on pre- and post-test achievement scores. Generally the results were very positive. Calculator counting activities (numeration) resulted in a significant improvement in mathematics achievement. Other activities including mirror cards (symmetry), body measurements (measurement), chip trading and base ten blocks (place value), and attribute blocks, colored cubes, and geoboards (patterns) also produced significant achievement gains.

Currently, the evaluation of the program in Puebla, Mexico is in the formative stage. The reactions of the children, the teachers, the administration, and the parents are very positive. The incorporation of the calculator and more recently the microcomputer has added to the high level of enthusiasm that the students have demonstrated for the activity-based approach to learning mathematics.

Bases upon the evaluation of the two project laboratory sites (Cincinnati, Ohio and Puebla, Mexico), the following recommendations regarding the use of calculators and other manipulative materials are suggested:

a) Math laboratory objectives need to be clearly defined in terms of integrating the use of the calculator with other manipulative materials. The activities for these objectives could introduce, support, supplement, or extend the on-going work on the regular classroom.
b) Math lab specialist and resource person should develop the the math laboratory activities and facilitate the instruction consistent with the school mathematics curriculum and currently used textbooks.
c) Math lab specialists should be encouraged to try innovative materials and methods and to focus on computation skills and expanded areas of basic mathematical skills, (e.g. problem, solving, estimation and approximation, geometry, measurement, constructing, reading, and interpreting tables, charts, and graphs).
d) The math lab specialist should be responsible for the evaluation and dissemination of effective instructional strategies involving calculators and manipulative materials. Math lab activities to reflect children's higher order congnitive skills and processes should be developed and used as assessment instruments.
e) Efforts should be made to encourage the continued use of calculators and manipulative materials in the intermediate grades. These students are still in the stages of developing abstract and logical thought processes and can benefit in their mathematical conceputal understanding with sustained use of materials.

1.1.3.3 Project "Keystrokes"

Arthur L White and Richard J Shumway,
The Ohio State University, Ohio USA

During the 1977-1978 school years a study was conducted by G Wheatly, R Shumway, T Coburn, R Reys, H Schoen, and A White on the impact of calculators on the learning of mathematics in the elementary schools in the USA. The research question was: "Will knowledge of basic facts, computational skill, or understanding of mathematics concepts be affected as a result of calculator use?"

The study included selected schools in the states of Indiana, Iowa, Michigan, Missouri, and Ohio. Two classes and their teachers at each grade level (2 thru 6) from each site participated. For each school, one class at each grade level used calculators and one class did not. A total of 1300 students and 50 teachers in 5 schools were involved. Calculators were used in the elementary school mathematics classes by 650 students and 25 teachers.
The results of this study show no evidence of a decline in mathematics learning as a result of calculator use. The results indicate that calculators extend the student's power in doing mathematics as well as improve their attitude towards mathematics. (For more details see WSC79).

During the study the research team observed teachers and the students in their uses of calculators. The insights gained in this process were put to use in the design and development of a series of calculator activities for grades kindergarten thru eight, entitled "Keystrokes". There are four "Keystrokes" books:

a) "Counting and Place Value" includes activities appropriate for grades kindergarten thru 3 and emphasizes number names, number sequences, place value, counting on, counting back, patterns, skip counting, and problem solving.
b) "Addition and Subtraction" includes activities appropriate for grade 2 thru 6 and emphasizes repeated addition, repeated subtraction, basic facts, patterns, problem solving, estimation, and applications through word problems.
c) "Exploring New Topics" includes activities appropriate for grade 4 thru 8 and emphasizes integers, decimals, estimation, problem solving, geometry, square numbers, and square roots.

1.1.3.4 Calculators Count

Russell A Langham, Senior Mathematics Adviser, Durham G.B.

In 1973 a pilot scheme was established by Durham Education Committee in a small number of secondary schools which enables teachers to investigate the potential use of electronic calculators in teaching mathematics. During the next three years a great deal of enthusiasm developed, as those involved slowly came to realize how useful the calculator was when used in a carefully structured way to support teaching. In brief, the teachers came to accept the calculator as a powerful tool which was valuable in teaching many aspects of mathematics. The children liked using them and there was some evidence that they were of value in developing understanding and motivation. The results of this investigation were published in 1976(see LAN76). This work was, however, exploratory by nature and it was recognised that there was still much to be done, particularly in providing more detailed information concerning those areas of mathematics in which the calculator can be used as a teaching aid and the manner in which it can be used.

As a result of these experiments a more detailed investigation was planned involving co-operation between several Local Education Authorities in the north east of England and the Schools Council agreed to fund the exercise. In 1979 the project entitled "The Use of Electronic Calculators in Secondary Mathematics" was launched with the following aims:

(a) To identify those areas of the secondary mathematics curriculum up to and including GCE 'O' level in which the calculator can effectively be used.

To identify effective classroom procedures for the use of electronic calculators in mathematics teaching.

In achieving these aims, the project was to concentrate on three aspect of calculator use, namely:

(i) a calculating aid
(ii) as a teaching aid
(iii) as an influence on curriculum development

(b) To produce guidelines and material for use by pupils and teachers.

The project was directed by R A Langham, Durhams's Senior Education Adviser. Colin Noble-Nesbitt, an experienced Comprehensive School, Head of Department, was seconded for seven school terms to act as a full-time project leader. The Shell Centre for Mathematical Education, a team of experienced Educational Researchers, agreed to act as consultants and co-ordinators to the work which was organized into three overlapping phases.

1 Planning and Investigation in Pilot Schools (terms 1-4)

The directing group, in consultation with the regional advisers and local teachers, identified and recruited the services of mathematics departments in ten pilot schools throughout the region, most of which had already experienced some use of the calculators.

These schools were provided with sufficient electronic calculators to enable one or two classes to have one per pupil, taking into account those already provided by their school board or the pupils themselves. The classes were selected to ensure that the full age and ability range was involved.

A working party was formed with a representative from each school, together with the directing group under the chairmanship of the project leader.

Using information provided by the northern mathematics advisers' review, the Durham pilot scheme and other work undertaken in the region, the working party produced a preliminary list of areas in the mathematics curriculum where calculators were relevant. Where possible, some guidance on how the calculators might be used was prepared so that teachers in the pilot school had a starting point from which to work.As investigation with calculators proceeded in the pilot schools, the experiences gained were carefully recorded and fed back to the working group at regular meetings. In this way, opportunity was provided for a thorough exchange of ideas. The working party then produced teachers' notes, work cards and other suitable resources for use in Phase II.

During this period the project leader visited each school on a regular basis, working with teachers and keeping an overall view of the progress made.

2 Trials (terms 4-6)

During this phase a second group of schools, with little or no experience of using calculators, were recruited to act as trial schools and arrangements were also made to involve some additional schools from Leicestershire. Information and materials produced by the working party were made availible to these schools which then undertook further trials and passed their comments and suggestions back to the working party.

3 Writing (terms 5-7)

During this stage of the project, the project team completed the final editing and preparation of the pupil materials and teacher guidelines, incorporating the results of piloting and trials with advice received from the evaluators and the schools council monitoring committee. The final result was a set of materials, designed to support the teaching of various aspects of mathematics found in most secondary school courses.

The materials consist of a collection of worksheets grouped under six broad topic areas, number concepts, sequences, series and limits, statistical investigations, errors and accuracy, geometrical and trigonometrical concepts and applications. They are designed as supporting materials in areas of the mathematics curriculum where the calculator can be used as a teaching aid in the development of concept understanding.

The emphasis throughout the project materials is on understanding. Careful planning has produced sets of exercises in which the student is led gradually to appreciate the meaning of the various concepts and to apply this understanding in the solution of realistic problems.

The student is often challenged to look at his answers, to search for patterns, to draw conclusions, to use what he has learned.

Excellent teaching approaches including all the elements recommended by the Cockroft Report (MAC82) are incorporated in the materials. Each topic area is introduced by teacher material which clarified the necessary mathematical background and often indicated possible introductory teacher exposition. Opportunities for fruitful discussion exist in abundance. Practical work is not neglected. Use of rulers, protractors, exercises on errors and accuracy. Problem solving, investigational work and applications to everyday situations are fully catered for. Skillful use of the materials by teachers will, undoubtedly contribute to curriculum development.

The materials are published (see CAL83) in loose leaf form with permission to reproduce 200 copies of any of the pupil materials without additional fees or permission. Thus one set will suffice for a complete school use.

1.1.3.5 **Project TIM**

Hartwig Meissner, Westf. Wilhelms-Universitat Munster, FRG

Since 1976 a research group called TIM (= Taschenrechner In Mathematikunterricht = calculators in mathematics education) has been working at the University of Munster, Federal Republic of Germany. The main question is to determine the possible impact of the calculator on computational skills and on the development of mathematical concepts. (For summaries see MEI73 and MEI83).

1 Mental Arithmetic

There is a general human behaviour always to choose the most convenient way for solving a given problem. This "economical principle" also can be observed regarding the use of calculators. If there is a calculator on my desk, why should I compute mentally? The calculator is more convenient because calculator results are quicker and more safe than mental arithmetic.
We started experiments to simulate that situation. All pupils of the class (grades 2 till 7) got a calculator on their desk. Solving their mental arithmetic problems (given on a work sheet) every pupil indeed used the calculator. But since they got many work sheets (each with up to 50 problems) the pupils step by step became acquainted with the problems and they started - while pressing the calculator keys - computing in their heads in parallel. More and more we observed pupils who already wrote down the result though they had not yet finished all the key pressings. They learnt that there are problems which they can solve without a calculator as safe but quicker.

Since working on work sheets is stupid and not very motivating, we varied the training frequently by using competitions. One or more groups of pupils used calculators, other groups computed mentally. The teacher called the problem and the quickest group with the correct answer got a score. At the beginning of these games every pupil wanted to be a member of a calculator group. But very soon they experienced to be quicker without the calculator.

Summarizing we can state (see AMV78, LAN79): The balance for the "economical principle" can be shifted towards mental arithmetic concerning multiplication/division table facts and concerning addition/subtraction problems with one or two digit numbers.

2 Paper and Pencil Algorithms

We also tested the "economical principle" in relation to paper-and-pencil skills. Low-achievers in grades 5 to 9 (age 11-16) got a calculator and worksheets with computational problems for the four basic operations.

They had to solve the problems as quick and a safely as possible. It was their own decision to use or not to use the calculator (which they were able to handle without any difficulties). The results of our case studies may be summed up as follows:

a) Almost no use of the calculator for problems like these:

$$\begin{array}{r} 39087 \\ \underline{+\quad 43256} \end{array} \quad \text{or} \quad \underline{46027 \times 6}$$

b) Calculator usage sometimes yes, sometimes no:

$$\begin{array}{r} 539082 \\ \underline{-\quad 268534} \end{array} \quad \text{or} \quad \begin{array}{r} 42805 \\ +\quad 73126 \\ \underline{+\quad 85007} \end{array}$$

c) Always calculators for the multiplication of two or more-digit numbers, for all division problems, and for the subtraction of more than two or more-digit numbers.

The pre-test was followed by a training of paper-and-pencil algorithms without calculators during 6 lessons. But there was no change in choosing the calculator in the post-test.
Conclusions: Our teaching should stress problems of Type (a) and (b) to maintain independence from the calculator in a certain degree. More emphasis in multiplication should be given in splitting up a more-digit factor into ones, tens, hundreds etc.

3 Syntactic and Semantic Use of Calculators

We use the calculator syntactically by pressing button after button according to the sequence of symbols given through the problem. Pupils often do not interpret the meaning of the display. They just take the result as a sequence of digits and therefore do not notice if they made a mistake (by solving a wrong problem or pressing wrong buttons).

However guess-and-test procedures with the calculator (e.g. guess if $\sin \beta = 0.4$) force the pupils to interpret the display for getting hints for a next better guess. To stress this semantic use of the calculator we emphasize the specific teaching method "one-way-principle" (see MEI79). The main idea is to see functions $x \xrightarrow{(f)} y$ as a unity (including the four basic operations, like $x \xrightarrow{+4} y$). We suggest that the three different problems

(i) given x and f, find y
(ii) given y and f, find x
(iii) given x and y, find f

are all approached using the same algorithm, i.e. the same syntactical sequence of calculator buttons. The first step is problem (i) just to learn the calculator sequence. Then the pupil uses this sequence combined with guessing and testing for problems of type (ii) or (iii). He intuitively interprets the wrong outputs by comparing them with the wanted output to get a feed back for the next inputs. Therefore the guess-and-test procedure develops an intuitive or relational understanding of the problem. This enables the pupils also to make better guesses without calculators, for example in daily life situations.

We did many case-studies using the one-way-principle. Some of them were done with calculator games. Others were done to find square roots, to teach percentages, or to introduce the trigonmetric functions.

4 Calculator Games

There are calculator games just for fun (e.g. "Upside-Down"). Others realize important mathematical goals like "Hit-The-Target", "The Big One", or "Factor Finding". The latter are guess-and-test games, which we used to train number sense and proportional reasoning and to develop an intuitive percentage feeling.

4.1 Hit-The-Target

Idea: Given is an interval $[x,y]$ and a number u. Find a second number v with $u \times v \in [x,y]$. The tables show protocols of the same pupil.

$\square \xrightarrow{\times 17} [560,585]$

Input	Output
20	340
200	3400
100	1700
50	850
25	425
40	680
30	510
20	340
25	425
30	510
35	595
31	527
32	544
33	561

Few number sense

$\square \xrightarrow{\times 37} [960,965]$

Input	Output
30	1110
27	999
26	962

Good number sense

4.2 The Big One

Idea: We use the constant facility to hide a $\div v$ -Operator. Which input gives the output 1, the big "one"?

$\square \xrightarrow{?} \square$

Input	Output
50	0,735...
51	0,75
52	0,764...
53	0,779...
60	0,882...
70	1,029...
65	0,955...
69	1,014...
66	0,970...
68	1

Few number sense

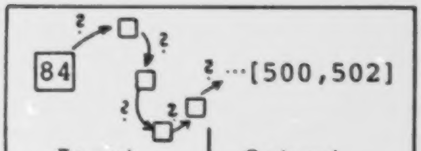
$\square \xrightarrow{?} \square$

Input	Output
50	0,735...
66	0,970...
68	1

Good number sense

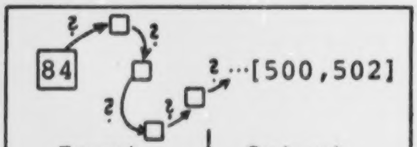
4.3 Factor Finding

Idea: Given is an interval [x,y] and a number u. find (like in Hit-The-Target) a second number v with $u \times v \in [x,y]$. If you miss [x,y] take the product $u \times v$ as the new number u_1 and find a new number v_1 with $u_1 \times v_1 \in [x,y]$. If you miss again take $u_2 = u_1 \times v_1$ etc.



Input	Output
× 6	504
× 1,1	554,...
× 1,01	559,...
× 0,9	503,...
× 0,9	453,...
× 0,99	449,...
× 1,1	493,...
× 1,01	498,...
× 1,01	503,...
× 0,995	501,...

Fairly good number sense



Input	Output
× 6	504
× 0,99	498,...
× 1,005	501,...

Very good number sense (Percentage feeling)

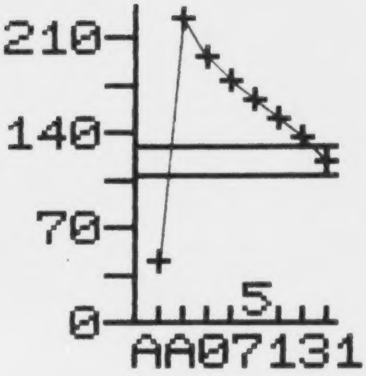
4.4 Results

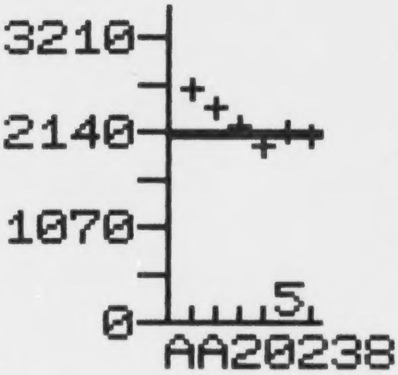
Many pupils invented their own strategies. Without being taught they intuitively and unconsciously developed effective procedures to solve the given problems with a remarkable small amount of guesses.

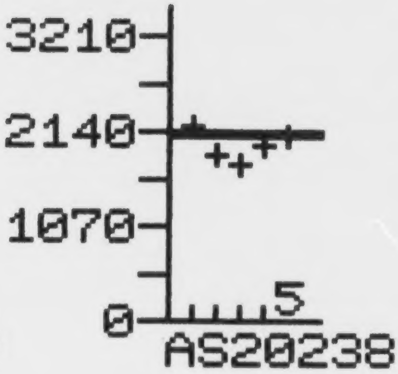
5 Developing Strategies

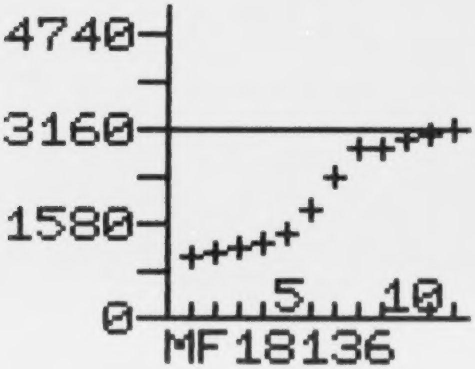
During the calculator activities we noticed that may pupils working semantically discovered strategies by themselves and that they reduce the number of trials effectively step by step. So we start a separate study (see MEI85). 33 pupils (age 10-13) had to work on Hit-The-Target. In individual interviews each child got a sequence of up to 50 tasks to solve. Each guess was recorded, 9 children also were monitored.

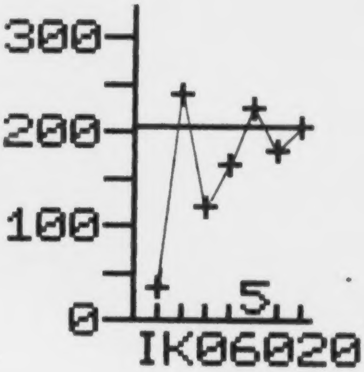
Though there are more than 1000 protocols we cannot give quantitative data. We transformed each guess-and-test sequence into one graph with the target interval, the number of guesses on the x-axis, a "+" for each guess value (calculator output), and a code number. It seems to be impossible to formulate general rules, but there are some typical patterns.











a) The first guess is an estimation problem. Some children realize that and get better during the test.

b) The first guess was too bad, let's make another guess (without using any information eventually hidden in the result of the first guess).

c) The first guessses sometimes only serve to find the right "ball park". The goal of the game (hit the target!) remains in he back, for instance the momentary strategy itself can be the goal.

d) There are systematic steps by twos, by tens or intuitive jumps (to accelerate or to reduce the "speed")

e) We observe monotonic approaches and alternating approaches.

f) We find children who do better during the test (i.e. need less trials) though the problems get harder.

6 Summary

The power of the calculator, getting correct results quickly and easily, gives three advantages for mathematics education. First we can use more realistic and therefore more interesting numbers without frightening out pupils. We rather can stress the structure of the problem since computation itself lost its difficulties. We can introduce new topics which were out of the ball park because of their computational volume.

Second the calculator allows a new working style. Since computation is no effort at all we can check our conjectures "just simply" without first looking for formulas or equations or mathematical theories to describe our problem. Systematical guess-and-test is a new approach to problem solving.

Third the calculator is a tool for concept building. In the same way as we use computers for simulations to train people to understand or to manage complex situations we can explore mathematical concepts. By means of the one-way-principle we discover the properties of functions and the relations within and between the domain and range. Applying this idea to the four basic operations the calculator also may help to develop a better number sense.

1.1.3.6 The ARK Project

Hans Brolin, Uppsala Universitet, Sweden

1 Introduction

In the spring of 1976 the National Board of Education appointed a group whose task would be to provide an analysis of the consequences of the use of the pocket calculator - known as the ARK group (from the Swedish "Analys as Raknedosornas Konsekvernser"). The ARK group's first task was to identify the problems and to launch various projects led by the group's members. (For more details see ARK84).

Three aspects of the use of the pocket calculator were investigated.

a) The handling of numbers in lower grades. All pupils should be proficient to perform by hand multiplication of double figures and simple divisions. In order to observe any deterioration and take appropriate measures, tests have been constructed to monitor any changes which may be attributed to the introduction of the pocket calculator.
b) Teaching methodology. Mathematics courses should be more applied, more experimental, more problem oriented, and more directed towards understanding of concepts.
c) The impact on curricula so far. Introduction of numerical methods and simulations into mathematical courses for age group 16-19. Introduction of applied statistics and critical analyses into mathematical courses for age group 16-19.

The following sections present a brief account of the most important subprojects undertaken wholly or partly under the auspices of the ARK group.

2 Subproject IAB (Ages 10-12)

Adolf af Ekenstam and Karl Greger

The Subproject IAB (Non-Algorithmic Basic Skills) was concerned mainly with the middle level of the comprehensive school (grades 4-6, age 10-12). Three reports and materials for the further education of teachers of this school level have been published.

a) Non-Algorithmic Basic Skills.
The mathematics taught at the middle level of the comprehensive school (age 10-12) is a complex mixture of skills, abilities and insight at many different levels of abstraction. It is, therefore, very difficult to anticipate the consequences which the introduction of calculators might have on this age group. The report ARK84 contains an analysis of such skills and insights which are independent of the means and methods used for routine computations involving the four basic rules of arithmetic (called: non-algorithmic basic skills), a test to measure these non-algorithmic basic skills and an analysis of the outcome of the test.

b) Children's Ability to Solve Mathematical Problems.
The ability to solve routine mathematical problems in the middle age-group of the comprehensive school (age 10-12) is already a very complicated mixture of many different skills and abilities. The report ARK84 describes an attempt to "define" and measure this ability. A test was designed and given to children in grade 6 (age 13). Observations were made and interviews carried out to obtain information on the thought paths followed by children engaged in solving mathematical problems. The construction of the test, the test results and the observations are discussed and analysed.

c) Children's Ability to Communicate Using Mathematics.
This report describes a pilot study of children's ability to communicate using mathematics. A test was designed to explore and measure some aspects of children's ability to communicate with or about mathematics. The test and its outcome are discussed and analysed. The main object of this investigation was to gain such insight into children's way of communicating with and through mathematics as to enable the authors to locate the areas in which further research needs to be done in this field.

d) Materials for Further Education of Teachers.
From the beginning all those engaged in the ARK Project realized that the very existence of electronic calculators and computers, whether or not they are, or will be adopted in school teaching, necessitates a critical analysis of the contents of the mathematics curriculum at all levels and of the ways and methods by which mathematics is taught. A group made up ofresearchers and practicing teachers set about producing materials for further education of teachers at the middle level, (for children of age 11-13), which would make teachers aware of the rapidly changing situation and so enable them to participate in and influence developments in a rational way.

The material produces so far is an analysis of the teaching methods and actual content of mathematics as taught to children of age 11-13, independent of the presently adopted curriculum. It is to be used in study groups.

3 Subproject RIMM (Ages 10-12)

Rolf Hedren

The RIMM project is a longitudinal study of the use of the calculator in grades 4-6 (ages 10-12). The object was to study the consequences of regular use of the pocket calculator where more complicated calculations are required. The following questions were posed: To what extent does regular use of the pocket calculator influence pupils' ability to do mental calculations and to perform calculations by hand? What about incentive and interest? In addition, we wished to establish whether pupils' understanding of numbers had improved or deteriorated.

a) Implementation

For the purpose of this project, specially constructed material was created to be used parallel with conventional teaching material. The latter excluded the use of pocket calculators. The new material was given a trial run and was then revised in accordance with received indications. The revised version was used for the experiment itself which began with grade 4 (age 10), 1979-80 and terminated with grade 6 (age 12), 1981-82. The trial material covered skills in multiplication tables and mental arithmetic, calculations by hand (fewer in comparison with standard teaching material), everyday problems in text form requiring a pocket calculator (where text problems were used, the pupils were given practice in estimations), and project work where the pupils themselves organized material and chose the form in which the results were to be presented (each such project was estimated to require 3-5 lessons).

In all, 8 trial classes and 3 control classes were involved in the experiment.

b) Method of Assessment.

Test and questionnaires were completed in all classes at the beginning of the autumn term in grade 4 (age 10) and at the end of the spring term in grade 6 (age 12). The test given in grade 6 was set by the IAB group, part of the ARK project, and is mentioned elsewhere in this report. Classroom observations were carried out and teachers were interviewed.

c) Results

The results for grade 4 showed no significant desparity between trial and control classes. As pupils neared the end of grade 6 trial classes showed better results than control classes in creative thinking, ability to estimate orders of magnitude, estimation, choice of correct method of procedure of problems in text form, and ability to extract relevant information in more complex problems.

Regarding other aspects covered by the experiment, no tangible differences between trial and control classes were noted. Results provided by the test in mental arithmetic and calculations by hand were interesting. Trial classes did as well as control classes in both tasks. The questionnaires showed:

> Trial classes tended to regard mathematics as being easier than did the control classes after the experiment. Trial classes did not appreciate the sections on estimations which were drilled rather intensively during the experiment. Control classes were considerably more sceptical towards the benefits of the pocket calculator in school both at the beginning and at the end of the experiment.
>
> Classroom observations of the trial classes provided some interesting results. Four classes were observed during 4-5 consecutive lessons, in general about 3 times per academic year. Many pupils preferred to calculate mentally or by hand even when they had access to a pocket calculator. On the other hand, some pupils used the calculator to excess to the extent of using it for estimations. Many pupils, mainly in grade 4, were nonplussed when the display was filled with figures. That when a new type of problem is introduced, the arithmetic in the introductory text should be sufficiently simple for the pupil to grasp the correct means of procedure. Problems which the pupil can solve mentally are to be recommended in this report.

From interviews with the teachers it was apparent that they were most enthusiastic about the trial. For one thing, it was felt that the use of the pocket calculator would introduce greater variety into mathematics teaching.

We can therefore draw the conclusion that regular use of the calculator in a teaching situation need not adversely affect the pupil's ability to calculate by hand or mentally as was feared. We have also established that the increased training in solving evryday problems, made possible by the calculator has imporved pupil's ability to estimate orders of magnitude. An imporvement has also been noted in the pupils' ability to choose the correct method of operation and to extract relevant information from text.

In addition to this, the trial has inspried new ideas for using the pocket calculator. It is to be hoped that these will lead to an even more effective training of the pupils' problem solving abilities than was possible during the experiment.

4 "Experimental Mathematics , Ordinary Course" (Ages 13-15)

Lars Aohman and Christina Bjorksten

Everyone needs to feel that his/her work is worthwhile. If such is not the case, life loses its lustre. The situation for pupils who lose interest in school work may be seen in this light. For such pupils, school has no incentive to offer. The school system then can be acused of failing in its duty which is to help young people develop in such a way that they can find a place in the society we have created. In particular, pupils who are less interested in theoretical subjects have been sadly negelected. These pupils need help to bring about a change in attitudes if they are to benefit from their school years.

With this in mind, we feel that there is a posssibility of bringing about a change in attitudes if mathematical studies can be pursued in a more practical way than is now the case. Pupils can themselves take problems from everyday life, from the reality which interests them. This will lead to the discovery that mathematics in all its forms surround them - an important discovery and one which renders the subject more interesting and exciting.

The pocket calculator has proved to be a great boon to those pupils who otherwise are forced to struggle with a calculation, never quite crossing the threshhold into discovery. Instead of a situation where the young people toil listlessly with endless and meaningless calculations, the opposite is possible. Young minds, confident of their abilities, expectantly searching out new areas to explore - wanting to know more - to learn! Should not school be more like this?

For several years now we have worked with "experimental mathematics with the help of the pocket calculator " and have compliled an "introductory pamphlet" which suggests practical problems suitable to start with. We have as well, a compendium of practical problems all of which have been tested in our own classes. Those who find this approach rewarding will hopefully be encouraged to create more material in this vein. There is also a program sufficient for a half study-day and an ITV program showing the practical approach and the pupils' response.

5 Subproject "Changes in Methodology in Mathematics for Natural Science/Technical Courses" (Ages 16-19)

Torgny Domar

The introduction of the pocket calculator into the classroom has radically affected the subject of mathematics. It is therefore imperative that both aims and methods in mathematics teaching be reviewed.

To begin with, where the calculator is used as an aid, it is more natural to expess the figures in decimal form. In my opinion, all solutions to mathematical problems should, as a rule be expressed in decimal form, the precise figure being given where appropriate. There will then be more opportunity (and hopefully inclination) to check the plausibility of an answer. (Nowadays this aspect of problem solving is sadly neglected.)

Now that we have the pocket calculator, problems can and should be constructed in such away that the figures do not easily lend themselves to calculations by hand. There is no longer either any need to weed out more complex, realistic figures.

In addition, with the advent of the pocket calculator, there is more opportunity to introduce practical steps in conjunction with the teaching of theoretical passages. Pupils can, for example, in connection with limits, series, probabilities etc, draw up various kinds of tables. This experience should facilitate the understanding of the theoretical aspects. The pupil should also be given the opportunity to allocate time for writing out as well as for working out the calculation. It is be hoped that the new emphasis placed on the practical aspects of problems will benefit some of the weaker pupils.

it is worth remembering that the pocket calculator does not only perform arithmetic. The type which should be in use on the Natural Science/Technical courses function as an easily accessible and easily manipulated set of tables. Important passages which were previously regarded as being too time comsuming can now be given the attention they deserve since the pupils can now manage more and more varied problems of this kind.

6 Subproject "Probability and Statistics Using Programmable Calculators" (Ages 16-19)

The aim of this project was to investigate how the use of programmable calculators might influence the teaching of probability and statistics in Swedish secondary schools. The project was organised as follows.

Preliminary teaching material was written and tried out in a secondary school in Gothenburg (Göteborgs Högre Samskola). On the basis of this experience, a complete textbook has subsequently been used in a large number of classes in different secondary schools in Sweden. Teachers and students been asked to fill in questionnaires regarding their response to the material.

The project has shown that programmable calculators can be used in a very effective and, for the students, stimulating way for teaching probability and statistics. With the aid of such machines important concepts and methods in these areas can be illustrated and investigated. Random experiments can be simulated, and variability and stability, inherent in such exerpiments, can be demonstated in a challenging way. Programmable calculators can also be used for a new methodology in these areas. Probability studies in this way will also provide the students with several non-trivial programming exercises.

7 Subproject "Project Work with Programmable Calculators and Computers" Ages 16-19)

Lennart Räde

The aim of this project was to write and test instructions for students who are to participate in mathematical projects using programmable computers or calculators. New teacher guide material was also included in the trials. Instructions for seven such project works have been written with the following titles:

Nikomachos and Euclid , Fibonacci, Prime numbers, Girl or Boy?, Stochastic geomety, Economy, Statistical data analysis.

8 Subproject NUMA (Ages 16-19)

Lars-Eric Bjork

From 1976-1980 experiments were conducted in numerical methods in mathematics and physics involving 16-19 year olds. The main objective of the NUMA experiment was to demonstrate how simple procedures like iteration, step method, extrapolation and simulation, in conjunction with the appropriate aids can provide extremely effective methods for prolem solving. Where programming is concerned, schools, depending on the type of equipment available, could concentrate on either programmable calculators or computers (BASIC). The aim has been to improve the pupils' understanding of mathematical concepts rather than that they should be preoccupied with the finer details of programming.

Included in the experiment were mathematics and physics. Objectives in mathematics were an introduction to numerical methods in connection with equation solving and integrals, numerical solution of differential equations, and probability theory and simulation. Objectives in physics were step methods in mechanics and electricity, step methods in connection with mechanical and electrical vibrations, and radioactive decay. Classes taking part in the NUMA experiment were given special problems in the central examinations.

During the experiment questions were posed: What type of programmable equipment (pocket calculator or minicomputer) is most suited for teaching mathematics and physics to pupils aged 16-19? What new steps of numerical nature should be included in mathematics courses for age group 16-19? Which areas in physics can be better understood through the use of repeated calculations in small steps? Do discrete models and step methods create a better understanding of basic concepts in mathematics and physics? Should not for instance numerical work with difference quotients and step methods in applied problems preceed the analytical treatment of derivatives, intergrals and differential equations? Is interest in mathematics likely to be stimulated by the practical steps introduced by numerical methods?

The findings of the NUMA experiment in mathematics are mirrored in the latest curriculum and in the text books for Natural Science/Technical studies.

9 "Applied Statistics for Pupils Ages 18-19"

Goran Anderson

Many mathematics teachers have expresed concern over certain passages in mathematics courses for Civis/Economics students. ("A cut-down verison of the mathematics course followed by Natural Science students with no heed given to the special need of those who study Civics or Economics".) Most criticism was directed at the section dealing with mathematical statistics at the end of the pupils' third year. This situation prompted the ARK project group to initiate a trial course where probability theory was for the most part substituted by applied statistics.

Text material for trial purposes was created around newspaper headlines which in various ways were connected with statistics. This material was to provide the basis for systematic training in the critical analysis of conclusions drawn from statistical data. The material provided examples of inappropriate or shifting definitions, inaccurate measurements, random errors, questionable sampling methods, confusion of cause and effect and biasing factors. The numerical examples were mainly concerned with non-response errors, the statistical margin of error for percent figures and the elimination of the influence of biasing factors (method of standardised averages).

The object of this experiment was to test the attitude of pupils and teachers to the use of a text of this kind in mathematics and also to establish to what extent the pupils can benefit by the contents. Critics have claimed that this type of material covers a difficult area which demands a higher degree of maturity than that possessed by this age group.

9 classes from 6 schools took part in the experiment which took placed at the end of the spring term of 1981. The following courses were represented: 3rd year Civics - 1 class, 3rd year Economics - 1 class, 3rd year Natural Science - 1 class, 2nd year Social Science - 3 classes. The 8 teachers involved cooperated on a voluntary basis. Out of consideration to the dissimilarity between the courses, variations in the method of administration were permmitted. 6 classes read all, or almost all, of the text consisting of 97 pages using about 20 lessons with the exception of the 3rd year Natural Science students who only used 12 lessons. Pedagogical methods also varied. In some cases, teachers led the lessons, in other classes pupils worked independently in groups.

It is, of course, hazardous to generalize the results from such a small-scale experiment as this. Some pupils reacted negatively towards the detailed text section. This was felt to be out of character with mathematics as a subject. On the whole, however, response was favourable. The majority of pupils thought that the course was interesting and that they had gained new insights and skills. Most teachers recommended that the critical analysis and applied statistics be extended over the entire course and were enthusiastic towards any possibility for cooperation with other subjects - particularly Civics.

This experiment has broken new ground in that it has shown that, under the conditions set by the experiment, it is possiblle to create a course which meets with the approval of the majority of pupils.

Perhaps the most important result achieved is to be seen in the proposed new syllabus for mathematics for Civics/Economics for 1984/85.

1.1.3.7 The GDR Project

reported by Hartwig Meissner, WWU Munster, FRG
based on the references F/W81, WAL82, FLA83, TAS84, F/W84 and on the personal exchange of experiences with project members

We will report on a long term investigation upon the use of calculators in mathematics education in the German Democratic Repulic (GDR). The project was organized, supervised and evaluated

BEST COPY AVAILABLE

by a calculator working group appointed by the national academy of pedagogical science in the GDR. The members of the working group were teachers, mathematics supervisors, and university teachers of mathematics education. They developed additional educational materials for the teachers of the experimental classes for almost all parts of the mathematics curriculum (grades 7-10). The concept of these materials was based on the following:

a) Despite the use of calculators children need skills and abilities in mental and written arithmetic. Mental arithmetic even will become more important specially for estimations and approximations, paper-and-pencil algorithms may be reduced in some degree. For example children should be able (see F/W84) to know the square numbers from 0 to 20, to know the sin-values for 0°, 30°,45°, 60° and 90°, always to check their calcualtions, and to do some of the tests (with easier numbers) without a calculator.
b) The pupils should learn how to round meaningfully results shown in the eight-digit display of the calculator.
c) The pupils should learn to check their results.
d) The calculator not only should be a computational tool but also and educational aid, for example for mathematical experimenting, or discovering processes.

In the four years study (Sept. 1979 - June 1983) more than 500 pupils in 21 classes were involved. At the beginning of the experiment (grade 7) each of these pupils (age 13-14) got a calculator. There were 18 control classes where the pupils did not get a calculator. The results of the study are based on written tests, on personal interviews wtih teachers and pupils and on classroom observations.

1 Computational Abilities and Skills

The pupils easily learnt to handle the calculator (in less than 4 lessons, in grade 7 the four basic operations, in grade 8 the memory keys). Computational mistakes were reduced dramatically. In comparison to written arithmetic the computational time was reduced to one third. Some of the children had difficulties with the exponential display or in using the memory (when and how to use the "hidden" number. They preferred writing down the intermediate results). Flow charts were introduced to describe the sequence of calculator steps.

1.1 Mental Arithmetic

There were two tests for mental arithmetic. In test A the pupils had to solve within 6 minutes up to 120 problems like 8+7; 12-3; 4x9; 64÷8; ... Test A was administered at the beginning of the experimental and at the end of each school year. The following table shows the numbers of correct results:

classes	start	grade 7	grade 8	grade 9	grade 10
experimental:	82	96	105	108	111
control:	80	98	105	112	112

In test B the pupils had to solve within 15 minutes 40 problems like $\sqrt{0.81}$; $2^2 + \frac{8}{\sqrt{4}}$; 0.2^2 ; $15 \div \frac{1}{2}$; $(-2)-(-9)$;

50 % of 300; $0.25^3 \times 4$; $(-0.4)^3$;... Also these results show that there will be no danger of a total dependence on calculators if there is an adequate concept of teaching. The table shows the number of correct results:

classes	grade 8	grade 9	grade 10
experimental:	30	35	35
control:	30	35	34

1.2 Paper-and-Pencil Algorithms

In Test C in grades 9 and 10 the pupils had to compute (without calculators) the following 10 problems within about 20m minutes (see F/W84).

a) 27.72 + 45.38 b) 31.73 + 572.4 c) 9.782 - 2.691
d) 23.41 - 4.201 e) 4.201 - 5.1 f) 374.29 x 233
g) 1.27 x 0.76 h) 648 ÷ 9 i) 85.23 ÷ 12 K) 76.4 ÷ 0.19

The following table shows the percentages of correct results:

classes	grade 9	grade 10
experimental:	69	75
control:	75	79

1.3 Algerbraic Terms

The pupils also had to learn analysing the structure of terms and the computing of terms mentally (without calculators), like 2 + 1.3.
The following table shows the percentages of correct results:
classes

Classes	grade 8	grade 9	grade 10
experimental:	64	89	86
control:	57	80	80

2 Conceptual Knowledge

Other tests for the grades 7-10 included 154 problems upon mathematical concepts anad abilities, some with computational demands (like determining the zeros of a quadratic function) and some without any computation like constructions or proofs). Comparing the results it was found:

	grade 10	grades 7-10
experimental better than control:	26%	38%
control better than experimental:	11%	14%
no difference	63%	47%

3 Problem Solving

The tests for grades 8-10 included 18 poblems like computing the volumes of solids, involving the Pythagoras Theorem or trigonomentric functions:

	grades 7-10
experimental better than control:	50%
control better than experimental:	6%
no difference	44%

An explanation for the better results of the experimental classes may be that those pupils had less difficulties in the computational part of the problem.

4 Summary

There are some results which cannot be described by test scores. For example the reduction of computational mistakes reduced the fear of mathematics in general, the self-confidence of the children improved. Specially in grade 7 the use of calculators was very stimulating for the classroom activities. But also in grades 9 and 10 mathematics gained in the view of the pupils.

1.1.4 Summary

1.1.4.1 Research Findings

Donna F Berlin, The Ohio State University at Newark, Ohio USA

There has been nearly a decade of research involving the use of the calculator in the mathematics curriculum in the United States. The result has been an extensive literature base marked by diversity (e.g. grade level, mathematics topic, type of calculator use, duration of use, and research design). Initial research endeavours focused upon surveys of both educators and non-educators. Many questionnaires and survey forms were developed and administered to a variety of populations across the country. Educators, particularly supervisors and teacher educators, were found to be more supportive of the increased use of the calculator in the classroom as compared to non-educators. Educators in the secondary school grades were more in favour of calculator use than those in the elementary school grades. Interestingly, many of these same educators were not actually using the calculator in their own classroom and expressed the need for more information regarding instructional uses of calculators (e.g., textbook calculator activities).

Non-educators were even less informed regarding the instructional use of calculators and primarily suggested that calculators should be used to check answers. Both educators and non educators, however, were in agreement over their concern that the use of calculators in the mathematics classroom would result in the loss of basic skills and the ability to do paper-and-pencil algorithms. Therefore, both groups recommended the use of calculators after the mastery of the basic facts and paper-and-pencil algorithms. Parents felt that the use of the calculator should be carefully "regulated" so as not to replace paper-and-pencil algorithms particularly in the elementary school grades.

It is not surprising to find that the majority of reseach on calculators has been directed towards the effect upon mathematics achievement. Research evidence does not support the fears expressed in the surveys. Mathematics achievement scores for students using the calculator have been as high or higher than for students not using the calculator (ROB80, SUY81, SUY82, SUY83). Findings regarding calculator use and mathematics performance can be summarized as follows:

The research evidence supports the use of calculators at all grade levels, indicating that achievement will not be harmed and may be enhanced when calculators are used. Furthermore, the evidence indicates that some mathematics content is taught better when calculators are used than when they are ignored (SUY82, see references)
The calculator research for the last half of the 1970's was directed toward finding the benefits of using calculators in the mathematics classroom. Attention was given to specific curriculum topics and specific learners. Some of the benefits associated with calculator use include: reduced memorization demands on children, expanded computational power, increased time and opportunity for higher level intellecutal processes, exploration of mathematical topics in more depth, and the opportunity to use practical, real life problems., The calculator can provide an element of motivation and interest in mathematics as well as a means toward greater individualized learning.

During the late 1970's concerns were still being expressed about the effects of calculator use in the classroom. These concerns include: the effect on motivation for the learning the basic facts and computational algorithms, an unhealthy dependence upon the technology, the assumption that the calculator can replace other mathematical manipulative materials, and the fear of broadening the gap between the have's and have not's (i.e., the issue of equal access).

Calculator research in the 1980's has revealed that the availability and use of the calculator has increased but the highest percentage of use still remains in the secondary school, particularly grade 12., Achievement studies continue to focus primarily upon the middle grades, grades 4 through 7, and grade 9, with some attention focused upon the benefits of using calculators with very young children, kindergarten and grade 1. Research in the 1980's has reaffirmed the positve effects on mathematics achievement associated with calculator use. Some of the specific benefits include facilitation of counting (i.e., number sense), facilitation of the understanding of subtraction and division particularly with low-achieving mathematics students, and the feasibility of using calculators wiht handicapped learners. An area of considerable research has been the use of calculators in problem-solving situations. During problem-solving activities the use of calculators tends to increase the students' accuracy in computations but tends to slow down the problem-solving process. It is important to emphasize the development and use of problem-solving strategies. Using the computational power of the calculator encourages the use of more realistic problems and directs attention toward the problem-solving process rather than the computations.

The research on calculator use and student attitudes toward mathematics has generally found no significant differences. These findings are contrary to the feelings expressed by teachers that calculators are motivational and promote a positve attitude toward mathematics. Perhaps the teachers perceive the students' positve attitude toward the calculators as an indication of their attitude toward mathematics. This does not appear to be the case.

A new direction in research seems to be appropriate based upon the previous findings. The question of whether to use calculators or of its effectiveness as a supplement to the mathematics curriculum should not be the major focus. Rather, a pedagogical, learner-oriented apporach to the role of the calculator needs to be explored. Calculator research parameters should be based upon cognitive, learning, and neuropsychological, theory and reserarch. Calculators should be integrated into the curriculum and the calculator research should be directed toward the facilitation of the learning process in order to improve instructional practice. The following areas for future calculator investigations are suggested:

a) Specific mathematical topics that would benefit from calculator activities.
b) The use of the calculator with other manipulative materials (multi-embodiments and mixed modes of representation).
c) The concrete to abstract continum and the sequence of learning activities.
d) The relationship of calculator usage to the cognitive developmental levels of the learners.
e) The relationship of calculator usage to the student's preferrred learning style.
f) The development and evaluation of calculator-integrated curriculum materials.
g) Long-term achievement and attitudinal effects of calculator use.
h) The development and validation of tests in which students make use of the calculator's capabilities.

1.1.4.2 A Proposal for Curricular Change

Grayson H Wheatly, Purdue University, Indiana USA
Paper presented as part of a Symnposium "The Effects of Calculator Availability on School Mathematics Curriuclum" at the Annual Meeting of the American Educational Research Association, San Francisco, April 9, 1979.

1 Historical Perspective

A careful study of the history of mathematics education will reveal that computation has always been the focus of the elementatry school mathematics curriculum. In the 18th century, children were taught ciphering. Rote computation was taught without any attempt to develop an understanding of the process. During the 19th century a

> few persons, like Warren Coburn, called for attention to meaning but the curriculum remained computational. During the 1930's there was a movement toward social utility and developing meaning in mathematics. Then in the period from 1958 to 1971 there was an emphasis on teaching the structure of mathematics. Viewed from the perspective of today, there was one unfortunate aspect of the so called "modern mathematics" movement. Much attention was given to rationalizing algorithms. Each of the complex algorithms (e.g. the division algorithms) was taught in great detail using a subtracting approach so that students would understand why the algorithm worked. But computational algorithms are nothing more than a set of rules for efficient paper and pencil answer-finding.

While it is important for students to understand the mathematics they are learning, time spent teaching children why algorithms work does little to help pupils understand mathematics and its applications. Vestiges of this unfortunate curriculum trend remain in current textbooks. Since 1971 there has been a well-defined swing toward the "Basics". In elementary school classrooms, this has been interpreted as more emphasis and time teaching basic facts and computation with even less time for concepts and applications. Throughout the eras described above, the curriuclum has remained computational. There is great contrast between the recommendations of leading mathematics educators and practices in the classroom.

At the same time this country was experiencing a Back-to-Basics movement in the 70's, advances in electronic tehcnology were reshaping the mathematical needs of society. Today the availability of inexpensive calculators has eliminated the need for complex computational algorithms. Before calculators (B.C.) it was necessary to be proficient in computations to apply mathematics. That has now changed. Cash registers show the amount of change and nearly every home and office has a calculator which can perform complex computations rapidly. It is quite ironic that as the calculator aviailibility was reducing the need for computation, schools were increasing their emphasis on paper and pencil computation proficiency.

2 Current Status of the Elementary School Mathematics Curriculum

A study of currently available elementary school mathematics texts suggests that the curriculum is computationally oriented. A typical fifth grade text devotes 139 pages (37%) to computation. However, teachers do not teach all the chapters. Most classes cannot finish the text in the time alloted for mathematics. In practice, the chapters ommitted are usually noncomputational. Chapters covering geometry, measurement, probability, statistics, and problem-solving are first targets of omission. The highest priority is given to teaching computation. In fact, while certain topics are ommitted, daily work is supplemented with additional computational worksheets. In many classes, computation completely dominates the year's work. My experience and that of my colleagues obvserving in schools almost daily for an entire school year suggests that computaion is the major emphasis in nearly every classroom. Calls for accountability and minimal competency testing have increased the time spent teaching computation since parents and school boards want to be sure that children know their "Basics". The newer texts reflect this trend.

3 The Proposal

a) Shift from a computation-based curriculum to a conceptually oriented curriculum utilizing the calculator as an instructional tool.
b) Elinimate the teaching of complex computations in the elementary school.

4 Rationale

a) We cannot afford the cost in time of teaching complex computations. Through surveying teachers, observing, and analyzing textbooks, I have concluded that of the first nine school years in studying mathematics more than two years are devoted to teaching the division algorithm. When the level of proficiency attained by the pupils is considered, along with this information, I conclude that we can no longer afford to teach complex long divison. An analysis of other computational procedures will reveal that the cost benefit ratio is far too high. The time saved by dropping this topic could be devoted to applications and prolbem-solving where the focus would be on when to divide, not how to divide.
b) Complex computations by paper and pencil are no longer necessary.
c) A computationally oriented curriculum inhibits the development of problem solving. A heavy emphasis on algorithmic thinking tends to encourage the application of rule-seeking behaviour, even when it is not appropriate. Successful problem-solvers need to try a variety of approaches, apply heuris tics, and recognize that more than rule identification is required. With a calculator to perform computation, pupils are free to focus on the problem rather than being distracted by switching to algorithmic thinking to compute.
d) The NAEP data suggest that 13-year-olds are not proficient in complex computations, even with all the time devoted. An examination of the National Assessment data (NAE78) reveals that less than two-thirds of the 13-year-olds could perform a long division problem (three digit by two digit). Only 45% of the 13-year-olds could solve a single-step word problem requiring single digit division.
e) Other topics are of more importance. In order to use mathematics meaningfully in today's society an understanding of certain other topics is important. More attention should be given to estimation, interpreting data, geometry, measurement, and the use of decimals and percents., Time will be avialable for these important topics only if computation is de-emphasized.

5 Recommendations for Curricular Change

Specifically, I am proposing that the following complex computations be excised from the curriculum.

a) Division with two or more digits in the divisior (e.g. 296 divided by 37).

This topic consumes more time than any other single computational topic in the elementary school curriculum. On the other hand, the single digit divsor algorithm is relatively easy to teach and could be retained. It is critically important that pupils learn the meaning of division. This can certainly be accomplished with single digit divisor computation. While I am recommeding that calculators be used to perform complex divisions, attention should be given to interpreting quotients and estimating results. It is essential that children attach meaning to their calculator results. Calculator activities have been published which focus on this objective (V/M77, KEY80).

b) Multiplication by two or more digits (e.g. 462 x 89). Children should gain proficiency with problems such as

$$\begin{array}{r} 326 \\ \underline{x\ 8} \end{array} \qquad \text{and} \qquad \begin{array}{r} 39000 \\ \underline{x\ \ \ \ 4} \end{array}$$

c) Addition of fractional numbers with unlike denominators. This topic has proven to be one of the most difficult and thus time-consuming topics in the curriuculum. The difficulty may result from the cognitive demands of the task. Addition of fractional numbers requires formal thought and most elementary school pupils have not reached this cognitive level. This skill can probably be taught more efficiently if begun in the junior high school.

d) Complex computations with decimals. Such tasks as 3.45 x 0.865 or 456.78|6.7 are more appropriately performed on a calculator.

6 What Should We Teach?

I will not attempt to describe a new curriculum in detail but instead make certain clarifying points related to the proposal.

a) Insist on memory of basic facts. It is improtant that children memorize the addition and multiplications facts. As Professor Schoen has shown, the calculator can be quite useful in teaching the basic facts.

b) Stress the concepts of addition, subtraction, multiplication, and division. It will be easier to do this if less emphasis is placed on computation.

c) Stress estimation and mental arithmetic. With calculators used for complex computations there is much improtance in children being able to estimate the answer, and to judge whether the result makes sense. Along with this, more time teaching mental arithmetic would be quite useful.

d) Emphasize applications. Mathematics is studied in large part becauses of its utility, yet the curriculum has not included much attention to applications for two reasons. We have reasoned that pupils need to learn to compute before they can apply mathematics and time has not been avialable because of the emphasis on computation. This must change.

e) Emphasize decimals.

f) Be sure that such topics as measurement, statistics, probability, and problem-solving are taught. These topics have appeared in elementary school texts but have been given low priority. They should be considered as basic.

g) Include computer literacy. Since computers play such a ce The NCSM list of 10 basics (NCS77) should become the foundation for curriculum develpment. Number five in the NCSM list is "appropriate computational skill". This paper suggsts one interpretation of this statement.

7 The Role of the Calculator

Although this proposal seems to stress the calculator as a substitute for paper and pencil computation, the calculator is actually a highly versatile instructional aid. A calculator can be used in concept development qutie effectively since many examples can be explored quickly. Consider the concept of prime number. A number can be tested for primeness quickly by dividing on a calculator. Pupils must know what divisors to try but the focus can remain on prime since the child does not have to take the time to perform the paper and paencil division. The conecpt of decimals can be introduced quite early and developed through calculator activities. More realistic numbers can be used since the calculator handles "grubby" numbers just as well as "nice" numbers. Many motivational calculator games develop important concepts such as place value, estimation, integers and functional rules.

8 Summary

The curriculum changes I am proposing may seem radical but the mathematics needs of society have changed in the last few years with the advent of electronic technology and will change even more in the near future. A computationally oriented curriculum is archaic and will not prepare our students for the 21st century. We must shift the emphasis to applications and topics that now are crowded out by the time spent on computation. Only by excising complex computations can time be made available for applications, problem-soplving, and other important topics. Such changes have far reaching implications, from teacher education to textbook preparation.

1.1.4.3 Results from ICME 5

Hartwig Meissner, Westf. Wilhelms-Universitat Munster, FRG

During the preparation of the Fifth International Congress on Mathematical Education (ICME 5) for Adelaide 1984 two of the 16 subtopics for the technology sessions were devoted to calculators. But only sixteen specialists from 12 countries came. They reported on national developments and summarized calculator activities and research results:

a) There has been much less research activity concerning calculators since ICME 4 (Berkeley) than there was between ICME 3 (Karlsruhe) and (ICME 4). (The main topics now are computers and problem solving.)
b) Research investigations since 1980 have not produced dramatic new insights or results, they mainly confirm the findings of previous investigations.

c) The overwhelming majority (over 95%) of investigations indicate that the use of calculators does not harm mathematics achievement in terms of the traditional curriculum and traditonal tests.

d) There are many countries with an availability rate of calculators to children of more than 80% where the school curricula ignore the existence of calculators or where the calculator use in mathematics education is not allowed for many grades.

There was a general agreement among the specialists, that the computational power of calculators is only one improtant aspect for mathematics teaching. Non-traditional calculator activities were presented to demonstrate the educational power hidden in calculators (guess and test, training mental arithmetic, exploration of mathematical concepts, etc.) An outlook was given how traditonal curriculum might change.

In the last session the group discussed future demands. Primarily there is no need for new teacher or student materials or for researach upon the effects of the calculator use in mathematics education. The experts have already done their job. The deep concern was that a large majority of the educational society still ignores the omnipotent existence of calculators. What will be the role of paper and pencil algorithms? What will be the value of teaching fractions? The objectives must be redefined!. Mathematicians, educators, administrators, text book writers - are you aware of your responsibility?

1.1.5 References

AMV78 Arens, E., Meissner, H., Vogt, Ch.: Taschenrechner in der Grundschule, Der Mathmatikunterrich 24, No 1/1978, pp 7 (FRG)

APM82 Queiques Apport de l'Informatique a l'Enseignement de Mathematiques. Publication de l'Associaton des Professeurs de Mathematiques de l'Enseignement Public (A.P.M.E.P.), No. 20, '82 (F)

APM83 Calculatrices 4 Operationes (Elementaire et Premier Cycle). Publication de l'A.P.M.E.P., No. 31, '83 (F)

ARK84 Bjorg, L.-E., Brolin, H.: The ARK Project - A Progress Report for the Period 1976-1983. Liber Utbildningsforlaget Stockholm 1984 (S)

B/G84 Belcastro, A., Guala, E.: Progettazione dell'Unita Didattica Calcolatori Tascabili nell'ambito del piano di lavoro didattico per la classe III del Progetto di Insegnamento Genovese. L'Educazione Matematica, Cagliari, Anno V, No 1, Apprile '84, pp. 39 (I)

CAL83 Calculators Count - Materials produced by the Schools Council Project Durham. Wright Printers, Dundee 1983 (G.B.)

CPS82 Calculators in the Primary School. The Open University Press, Walton Hall, Milton Keynes 1982 (G.B.)

EDW82 Edwards, Allen: The Hand Held Calculator in a Third World Country. Numeracy Project of Papua New Guinea, 1982 (ERIC # ED 222381)

EDW84 Edwards, Allen: Computational Estimation for Numeracy, Educational Studies in Mathematics , 15, (1984), pp. 59 (ERIC # EJ 293951)

GPR83 Guala, E., Parenti, L., Repetto, I., Zappa, A.: Un'esperienza di aggiornamento su possibili usi di strumenti di calcolo rivolta a Insegnanti di Scuola Meida. L'Insegnamento della Matematica e Delle Scienze Integrate, Paderno Del Grappa, Quaderno No. 7, supplemento Vol. 6, No. 4, Agosto '83 (I)

H/V82 Houben, J.P., Vanhamme, W.: Les calculatrices dans le cycle inferieur. Mathematique et Pedagogies, No. 39, '82, pp 91 (F)

IRG81 Calculatirces de Poche et Enseignement des Mathematiques. Activithemes pour la classe de seconde, IREM de Grenoble, '81/82, pp. 3 (F)

IRR83 Calculettons, IREM de Renners. No. 6, Jun. '81, No. 9, Apr. '83, No. 10 Jul. '83 (F)

KEY80 Reys, R.E., Bestgen, B.J. Coburn, T.G. Marcucci, R., Schoen H.L., Shumway, R.J., Wheatley Ch. L. and G. H., White, A.L.: Keystrokes - Calculator Activities for Young Students. (4 books). Creative Publications, Palo Alto 1980 (USA)

KNO80 Knopf, Peter, Der Taschenrechner in der Schule - Probleme, Forschungsergebnisse und didakitsche Ansatze. Schweizerische Koordinationsstelle fur Bildungsforschung. Aarua 1980 (CH)

LAN76 Langham, Russell A. (Ed.): The Use of Electronic Calculators in Secondary Schools. Country of Durham Education Committee. Offset Litho at Darlington Teacher's Centre 1976 (G.B.)

LAN79 Lange, Barbel: Schneller Kopfrechnen mit dem Taschenrechner. In "Sachunterricht und Mathematik in der Primarstufe," Aulis Verlag Deubner Koln 7/1979, pp. 430 (FRG)

MAC82 Mathematics Counts' (The Cockcroft Report). HMSO London 1982 - ISBN 0112705227 (G.B.)

MEI178 Meissner, Hartwig: Projekt TIM 5/12 - Taschenrechner im Mathematikunterricht fur 5- bis 12 Jahrige. Zentralblatt fur Didaktik der Mathematik 10 (1978), pp. 221 (FRG)

MEI79 Meissner, Hartwig: Problem Solving with the One Way Principle. Proceedings of the 3rd Internaitonal Conference for the Psychology of Mathematics Education, Warwick 1979 (G.B.)

MEI83 Meissner, Hartwig: The Effect of the Early Use of Calculators on the Acquisition of Number Concepts and Skills. Proceedings of ICME 4 (Berkeley 1980), pp. 605. Birkhauser Boston 1983 (USA)

MEI85 Meissner, Hartwig: Selfdeveloping Strategies with a Calculator Game. Proceedings of the 9th Internaional Conference for the Psychology of Mathematics Education, Utrecht 1985 (NL)

MER84 Mathematical Education Review 1980-1984. the Shell Cente for Mathematical Education, Nottingham 1984 (G.B.)

MIN79 Ministero della pubblica Instruzione: Programmi, orari di Insegnamento e prove di esame per la Scuola Meida. Gazetta Ufficiale No. 50, 20.02'79 (I)

MUR84 Mura, Roberta: Stude comparative de l'utilisation spontanee d'une calculatrice. Canadian Journal of Education 9:4(1984)

NAE78 Carpenter, T., Coburn, T., Reys, R., Wilson, J.: Results from the First Mathematics Assessment of the Naitonal Assessment of Educational Progress. NCTM Reston Virginia 1978 (USA)

NCS77 Positionpaper on Basic Skills, NCSM 1977 (USA)

NDE83 Noel, Guy: Quelques emplois du calcualteur programmable dans une classe. Proceedings of ICME 4 (Berkeley 1980), pp. 619. Birkhauser Boston 1983 (USA)

RIC79 Progetto RICME (Finnovamento del Curricolo Matmatico Elemetare). A. Armando, Roma, '79/784 (I

ROB80 Roberts, D.M.: The Impact of Electronic Calculators on Educational Performance. Review of Educational Research 50, (1980), pp. 71 (USA)

ROM84 Romanovskis, Tomass; Micro Calculators (in Russian). Riga Zvaigzne 1984 (USSR)

ROS82 Rossi, A. L Il calcolatore tascabile come strumento didattico. Dell'Acqua in Annali della Pubblica Istruzione, 1982, pp 699 (I)

SUY79 Suydam, Marilyn N.: Working Paper on Hand-Held Calculators in Schools. Prepared for the Second International Mathematics Study. Columbus Ohio 1979 (USA)

SUY81 Suydam, Marilyn N.: The Use of Calculators in Pre-College Education - Fourth Annual State-of-the-Art Review. Calculator Information Center, Columbus Ohio 1981 (USA)

SUY82 Suydam, Marilyn N.: The Use of Calculators in Pre-College Education - Fifth Annual State-of-the-Art Review. Calculator Information Center, Columbus Ohio 1982 (USA)

SUY83 Suydam, Marilyn N.: Calculators - A Categorized Compilation of References. ERIC Information Analysis Center for Science, Mathematics, and Environmential Education. Columbus Ohio 1983 (USA)

T/884 Tyler, Ken and Burkhardt, Hugh: Calculator Maths (6 volumes). The Shell Centre for Mathematical Education, Nottingham 198-3 (G.B.)

TOR82 Torcq, Daniele Calcul numerique et calculatrices, Activites Mathematiques au College. E.M. de Picartdie, '82 (F)

UMI82 Calculatori per la scuola. Edited by Unione Matematica Italian (UMI), Assoziazione Italiana Cocolo Automatico, Applicazioni degli Elabora tori nella Didattica (AICA/AED). Notiziario dell'UMI, Nov, 83, Anno X, supplemento al No. 11 I

VAN78 Vanhamme, W. : De la presence des calculartrices dans la classe a la decouverte de notions mathematiques (1er cycle secondaire). Les Calculatrices et l'Enseignement des Mathematiques, Proceedings of the Commission International pour l'Etudes des Mathematiques (CIEM), Luxembourg 1978

V/M77 Vervoort, G., Mason, D.: Calculator Activities for the Classroom. Copp Clark Publishing, 517 Wellington Street West, Toronto Canada 1977

WAL84 Walsch, Werner (ed.): Taschenrechner in der Schule. Martin-Luther-Universitat Halle-Wittenberg 1984 (GDR)

WSC79 Wheatley, G.H., Shumway, R.J., Coburn, T.G., Reys, R.E., Schoen, H.L., Wheatley, Ch. L., White A.L.: Calculators in Elementary Schools. Arithmetic Teacher 27 (1979/80), pp. 18 (USA)

BEST COPY AVAILABLE

W/W80 Wynands, A., Wickman, D.: Rechenfertigkeit und Taschenrechner - Istzustand 1979/80. Zentralblatt fur Didaktik der Mathmatik 12 (1980), pp. 162 (FRG)

WYN84 Wynands, A.: Rechenfertigkeit und Taschenrechner - Ergebnisse einer dreijahrigen Untersuchung in den Klassen 7 bis 9. Journal fur Mathmatikdidaktik 5 (1984), pp. 3 (FRG)

1.3 TECHNOLOGY AND THE SECONDARY SCHOOL MATH CURRICULUM

Sheila Ahmed. UK
Robin Burgess, Australia
John Callaghan. Australia
Kevin Doyle. South Africa
John Egsgard. Canada
James Fey USA
Bill Forbes. Australia
Kevin Ford. Australia
Michael Gray. Australia
Clive Healey. Australia
Patrakoon Jarivavidyanont. Thailand
Stephen Keegan. Papua New Guinea
Ian Kinnersley. Australia
Tony Lantry. Australia
Charles Lovitt. Australia
Paul Luscombe. Australia
Masahiro Makino. Japan
Christine McCabe. Australia
D.F. McKenzie. New Zealand
Julianne Murphy, Australia
Elizabeth Remilton. Australia
Sergio Ruffini. Australia
Jim Swift. Canada
Tsuneo Uetake. Japan
Chair/Reporter: James Fey USA

1.3.1 Introduction
1.3.2 Improved Teaching of Traditional Maths
1.3.3 New Mathematical Content and Priorities
1.3.3.1 Realistic Problem Solving
1.3.3.2 Emphasis in Understanding
1.3.3.3 The Algorithmic Point of View
1.3.3.4 Graphic Display Methods
1.3.3.5 New Approaches in Geometry
1.3.3.6 Statistics
1.3.3.7 Discrete Mathematics
1.3.4 Teacher Education
1.3.5 Summary

1.3 TECHNOLOGY AND THE SECONDARY SCHOOL MATHEMATICS CURRICULUM

Sheila Ahmed. UK
Robin Burgess, Australia
John Callaghan. Australia
Kevin Doyle. South Africa
John Egsgard. Canada
James Fey USA
Bill Forbes. Australia
Kevin Ford. Australia
Michael Gray. Australia
Clive Healey. Australia
Patrakoon Jarivavidyanont. Thailand
Stephen Keegan. Papua New Guinea
Ian Kinnersley. Australia
Tony Lantry. Australia
Charles Lovitt. Australia
Paul Luscombe. Australia
Masahiro Makino. Japan
Christine McCabe. Australia
D.F. McKenzie. New Zealand
Julianne Murphy, Australia
Elizabeth Remilton. Australia
Sergio Ruffini. Australia
Jim Swift. Canada
Tsuneo Uetake. Japan
Chair/Reporter: James Fey USA

1.3.1 Introduction

Computers and other electronic information technology are becoming standard problem-solving tools in business, industry, government, and personal affairs. Thus secondary schools are being challenged to rethink the content objectives and instructional approaches of all curricula-especially in mathematics. AT ICME 5 in Adelaide, a theme group with representatives from 10 countries examined three dimensions of potential technology-induced change in secondary school mathematics curricula:

(a) Possibilities for improved teaching of traditional mathematical topics.
(b) Opportunities for new selection and organization of mathematical topics - new priorities among traditional concepts and skills, new topics from computer-related discrete mathematics, and access to important mathematical ideas that have previously been too difficult to teach effectivley.
(c) Problems of teacher education and curriculum change presented by the technology challenge.

On each issue the theme group memebers sought a survey of outstanding work from around the world, analysis of the strengths and weaknesses in that work, and recommendations for needed research and development work.

The extent and style of classroom use of computers varies widely - even amoung countries with similiar educational systems and similar access to technology. Furthermore, the emergence of low-cost and high-powered computing technology is such a recent phenomenon that the state-of-the-art in its use for education is changing almost daily. Thus the following report represents only a limited survey of a rapidly changing field and, more importantly, an attempt by thoughtful mathematics educators to set an agenda of challenges for future development.

1.3.2 Improved Teaching of Traditional Mathematics

Developments in computer-enhanced teaching of traditional mathematical skills and concepts fall into several major strategy categories. First, many teachers believe that students who write computer programs to guide mathematical calcualtions will, in the programming process, acquire unprecedented insight in to the structure of relevant concepts and skills. For instance, in a project in the West Susex Institute of Higher Education in the United Kingdom, students are learning impressive amounts of geometry and trigonmentry in the process of writing progams to create computer displays of two- and three-dimensional objects. A project based at Asia University in Japan has approached the teaching of elementary ideas in number theory by having students analyse and modify programs that search for factors, multiples, and prime numbers (apparently without accesss to classroom computers!).

A second common form of computer-enhanced instruction uses pre-programmed packages to provide students with tools for numerical or graphic exploration of mathematical ideas. For instance, the Reality in Mathematics Education (RIME) project in Australia provides lessons involving classroom problem solving, with accompanying computer programs to help in the search for number patterns and problem solutions. In many countries teachers are beginning to use function graphing utility programs to provide an electronic chalkboard for algebra or calculus classes.

The interactive capabilities of computers have encouraged development of a third stragey to enchance mathematics teaching. Vivid simulation and game formats embody problem solving situations that lead to discovery of new concepts and skills. Work at the Shell Centre in England and at the Unversity of Illinois in the U.S has produced outstanding examples of the potential in such computer uses.

Despite the exciting promises in the above instructional innovations, made possible by computers, the most common form of computer use to enhance mathematics instruction is some form of drill and practice of traditional skills. Many of these programs do present such practice in a new style that is far more attractive than pages of exercises in standard texts. The most sophisticated are beginning to provide insightful diagnosis of student learning difficulties. However, the various reports of "what's happening" in computer-based mathematics instruction included few references to the sort of comprehensive computer-assisted learning systems that would provide and manage mathematics education as teacher substitutes.

With a few exceptions in each of the most technologically advanced countries, production of computer-based instructional aids is still a "cottage industry". Individuals or small groups conceive and progam ideas for a market place that lacks essential communication networks for sharing ideas and provides too little support for in-service education to help teachers learn about and take advantage of the emerging software. Most analyses of computer potential to enhance mathematics teaching describe new classroom dynamics in which teacher and students work with the computer to study mathematical ideas and to solve mathematical problems. This new instructional reality will not occur without careful nurturing.

Thinking about actions that would be most useful in helping schools exploit the potential of computers to improve instruction in traditional content areas, the ICME theme group made the following recommendations:

(a) Software packages should emphasize things that teachers cannot normally do without computer assistance.
(b) Each country or state should have libraries of carefully evaluated software and teaching ideas that make use of that software. Communicaton networks for dissemination of that information are equally important.
(c) One very useful component of such a software library would be a collection of mathematical sub-routines which teachers could draw on to constuct their own personalized classroom software.
(d) To overcome the substantial financial risks involved in production of high quality educational software. It seems advisable for governments to provide seed money to meritorious development ideas.
(e) Government agencies can also provide an important service by supporting, or at least coordinating, research to study the broad question of effectiveness of computer aids in teaching and the value of specific strategies in particular.
(f) Efforts should be made to minimize the problems of machine incompatibility - either by thoughtful design of the software itself or by standardization of hardware requirements.
(g) The introduction of computers to mathematics education brings an opportunity, but also a risk of losing very capable teachers to the computer studies curriculum. It is essential that efforts be made to retain capable teachers in mathematics, so that the subject is not taught only by those who are computer-reluctant.

While there are striking examples of effective computer use in mathematics instruction, it is clear that schools around the world have barely begun the effort to captialize on this impressive new resource.

1.3.3 New Mathmatical Content and Priorities

For those who have thought about and worked on using computers to enhance teaching of traditional mathematics, the next big task is analysis of ways that computing implies change in the content and emphasis of current mathematics curricula. The near universal access to calculators and computers that is emerging in many countries offers a broad range of opportunities to reorganize and redirect the priorities of traditional courses. It seems likely that some old favourite topics will decline in importance as computers take over routine facets of mathematics. Other topics which, despite their importance in applications, have been inaccessible to secondary school students due to computational complexity, now make a legitimate claim for space in the curriculum. Still other topics in mathematics have gained new stature because of their importance to the new discipline of computer science.

Participants in the ICME technology theme group suggested that in assessing the validity of proposed curricular changes it is important to keep three questions in mind:

* What are the requirements of mathematical literacy in a highly technological society?
* What is the essential mathematical basis of technololgy, engineering, science, and the computer and information science?
* What mathematical content and instructional approaches are essential to develop student creativity and problem solving abilities?

The following points describe representative ideas that are being studied. They do not exhaust the possiblities, and the order of presentation here does not imply any priority of significance.

1.3.3.1 Realistic Problem Solving

One of the most attractive and least controversial implications of technology in mathematics curriculum is the introduction of application material that has realistic computational complexity. No longer must problems be limited to simple equations with one or two variables and simple rational number solutions. Time devoted to practising various computational skills now, due to the ubiquity of calculators and computers, can be reallocated to the study of the mathematical modelling process. At present, calculators are used only with reservations in many classrooms. They offer promise as powerful tools in extending the scope of standard courses - particularly for less mathematically able students. Computers offer similar prospects in advanced topics.

1.3.3.2 Emphasis on Understanding

Although calculational powers of technology seem to argue for diminished attention to student skills, understanding of fundamental concepts remains essential.. Effective problem-solving requires comprehension of ideas so that students can select and interpret the results of appropriate computational work. While such objectives are always expressed in the goals of mathematics courses, it now seems possible to make much more substantial progress toward them.

1.3.3.3 The Algorithmic Point of View

If computers are to be useful tools in solution of mathematics problems, it is very often necessary for the computer user to devise an algorithmic procedure for carrying out required computations. Ability in this broad problem-solving heuristic can only develop if students are frequently asked to devise algorithms for mathematical processes and to analyse the generality, accuracy, or efficiency of various algorithms for a single objective. Thus it seems desirable to develop and explore the effects of curricula which emphasize such an algorithmic view. There are many opportunities for the algorithmic approach in algebra, geometry, and analysis - each with particularly attractive features. For instance, the evaluation of polynomials can be accomplished in at least two basic ways of vastly different computational desirability. One example involves 3 addititons and 7 multiplications. When rearranged it requires only 3 additions and 3 multiplications. As another example, the various methods of solving systems of linear equations offer quite different features of generality, efficiency, and sensitivity to error.

1.3.3.4 Graphic Display Methods

One of the most striking contriubtions of the microcompter revolution is the provision of high resolution, dynamic, colour graphic displays for mathematical relations. This new tool for assisstance of mathematical thought offers an invitiation to reconstruct the treatment of traditional topic aresa. For instance, the algebra of polynomial, rational, exponential, and trigonometirc expressions has commonly been driven by an array of symbol manipulations procedures aimed at discovering a single "unknown" number. With computer graphics it becomes relatively easy to construct an accurate picture of the total relation given and to use visual reasoning in answering quantitative questions. This power can be extended readily to vivid presentation of multivariate functions like $z = \sin(x+y)$, previously judged inaccessible except to more advanced students.

1.3.3.5 New Approaches in Geometry

Computer graphic capabilities also have implications for curricular reorientation in geometry and trigonometry, calling on new ways of intuitive and formal reasoning in that subject. At least three areas of striking impact hold promise.

The dynamic graphics of computer displays offer a natural medium for exploration of geometry through a transformation or motion perspective. The computer helps students to visualise effects of various standard motions. Its application to computer-aided design gives a strong motivation for study of geometry. The effects of changing shape parameters and viewing perspectives can be readily explored with computer assistance in an interactive mode.

The new computer graphic power should also lead to enhanced student spatial visual abilities. A more concrete approach to the analytic geomtery of 2- and 3-dimensions (including a 'physical' approach to vectors), and a more ready access to the geometry of students' physical space (the earth's surface, space mathematics, and the many practical applications of trigonometry). Each of these important topics has often received too little attention because of difficulties presented by the 'chalk board' display and communication medium that dominates traditional instruction.

Yet another stiking computer innovation - the Logo language and its associated 'turtle' graphics - brings impressive new opportunities to school geometry. The turtle provides a structured drawing tool which yields insight into geometric figures through student explorations. At the same time students meet the difficult ideas of recursion, iteration, and differential geometry in a concrete and visually appealing format.

1.3.3.6 Statistics

Current teaching of statistics is bound by several constraints which make it an unattractive school subject. Examination syllabi are based on outdated conceptions of the subject, and sets that are suitable for analysis by student arithmetic skills. The use of computers in statistics and its teaching promises a break from both limitations.

First, the capacity of computers for dealing with vast collections of data and for analysing and displaying that data in a variety of revealing ways is reshaping the structure of statistics as a discipline. The traditional path from combinatorial counting to discrete probability to random variable and standard distributions is no longer the only viable entry to statistical concepts and methods.

Second, availabiltiy of computational assistance for students makes it possible to explore realistic data that is of interest to students. This new realism offers prospect of providing genuine understanding of the principal activity of statisticans, namely the exploration of data and discovery of information it contains. Computers also facilitate exchange of concepts like variability and distribution, and a new range of graphic displays. Thus the computer makes an increasingly important aspect of mathematics accessible in ways that have never been available to school instruction.

1.3.3.7 Discrete Mathematics

At the same time computers help teaching of traditional mathematics and change priorities among those topics, they offer an new agenda of mathematical ideas and methods uniquely suited to computation. The discrete iterative approaches of difference equations, linear progamming, and numercial approximation methods in calculus take on new importance in today's computer-rich environment for quantitative analysis. Furthermore, those who will become specialists in the design of computer hardware and software systems needs to learn aspects of mathematics like graph and network theory, algebraic sturctures, combinatorics, and logic that have not been part of the traditional mainstream leading to calculus.

At this time work has only begun on the opportunities and challenges described above. It seems clear, however, that major changes in traditonal curricula are on the way.

1.3.4 Teacher Education

To meet the challenge of computing in mathematics education the crucial change agents will be classroom teachers. At the present time mathematics teachers may be classified, according to their level of interest and knowledge, as:

Computer Illiterate -
teachers who have rarely used a computer, are unaware of its potential in the classroom, or are intimidated by the technology.

Computer Literate -
teachers who are familiar with the operation of a computer and may use the computer extensively for programming and related activties. Such teachers commonly regard the computer as an object of study and often refuse or choose not the use the computer as an adjunct to the teaching process.

Computer Conscious -
teachers who are able to integrate the computer within a lesson and adopt a flexible approach to its use. This is the stage to which we would try to elevate all mathematics teachers.

While a variety of practices exist for ecducating teachers in the integrated use of computers, each method has some shortcomings. An in-school facilitator is often effective, leading by example. But is some settings this same peer example may face limited acceptance due to the scepticism of equal-status peers. On the other hand, visiting experts bring a kind of official status with their ideas, but commonly have too little time for the kind of extended assistance and follow-up required for real change. Short courses taken outside the target school can be effective and efficient dissemination strategies. However, they fail to attract teachers with out-of-school commitments and thus tend to reach only those who are already enthusiasts. A variety of in-school retraining courses currently seem to concentrate on computer literacy and reach only a small proportion of the teaching population. Formal post-graduate courses have similar limitations as approaches to large numbers of teachers or as innovation strategies that address local school needs.

The effort to infuse mathematics teaching with appropriate computer use also faces several other barriers. At the present time, in-school computer facilities in many countries are dominated by computing studies teachers and students. There is a lack of information about and evaluation of appropriate available software. And teachers have very little spare time in their already hectic school days to take on the formidable task of learning computing and its uses.

In search of strategy recommendations that meet the major problems of teacher education identified above, the ICME theme group participants were impressed with a model due to the RIME project in Australia. That approach develops a team of 'experts' who travel to interested schools providing demonstration lessons in the normal school environment. The demonstration phase is normally followed by implementation efforts of the teachers in the target school, with support of the RIME staff when needed. An essential feature of this strategy is provision of relief from teaching responsibilities for the teahers who are expected to develop new teaching skills and to put them into practice.

1.3.5 Summary

The school response to opportunities for computer use in secondary mathematics has only begun to be realised, in even the more advanced and active countries. There are a few impressive prototypes of potential. But a great need remains for development of effective computer software, alternative curri culum materials, and in-service education of teachers.

The ideas in this paper were contributed by 24 people who are now joined in an information sharing network under the leadership of Sergio Ruffini, P.O. Bos 224, Melton, Victoria, 3337, Australia.

1.4 Symbolic Mathematical Systems and Their Effects on the Curriculum

Alex Angus. Australia
Les Berry. Australia
Stan Clark. Australia
Anthony C Hearn. USA
Sid Morris. Australia
Mike F Newman. Australia
Alexander Ollongren.
The Netherlands
Gerald Porter. USA
Jim Richardson. Australia
Bruce Stack. Australia
Linda Tame. New Zealand
Kay Wellington. Australia
Marc Bergman. France
Bruno Buchberger. Austria
Erwin Engeler. Switzerland
Pertti Lounesto. Finland
Haruo Murakami. Japan
Margaret Northcott. Australia
Page Warren. USA
Raymond Parker. South Australia
Barbara Reilly. New Zealand
Frederick G Schmitt. USA
David Stoutemyer. USA
Dennis Trenerry. Australia
Ron Wenger. USA
Chair/Reporter: Bruno Buchberger. Austria

1.4.1 Introduction

1.4.2 Abstracts

1.4 Symbolic Mathematical Systems and Their Effects on the Curriculum

1.4.1 Introduction

At ICME 5, a group of approximately 30 persons worked in 4 meetings on "Symbolic Mathematical Systems and Their Effects on the Curriculum". Among the participants were people actively involved in research in computer algebra and symbolic software systems as well as high school and univeristy teachers interested in the possible use of such systems for mathematical education.

After a brief introdcution into computer algebra (symbolic mathematics, symbolic computation, analytic computation ...), which was illustrated by demos of muMATH and some other software systems, desirable impacts of present and future symbolic mathematical systems on mathematical education were discussed. The papers presented on the next pages served as preparatory and reading material for these discussions. They cover a wide range of possible opinions, proposals, reports on experriments etc. These papers were not refereed (in order to conserve their character as contibutions for the discusiion) but edited. (Editors: B Buchberger and D Stoutemyer). For these procedings, most of these papers had to be shortened. (B Buchberger alone takes the blame for all insufficienies caused by this shortening. Due to a strict deadline the shortended versions could not be approved by the authors!). The full versions of these papers will appear in the next issue of the SIGSAM Bulletin. [Consequently the Editor of the Role of Technology Report has published only the abstracts/introductory or summary sections of the papers referred to above].

Possilbe effects of symbolic mathematical systems on all types of curricula were discussed: academic math education, computer science education, math education of physicists etc., high school math education, preservice teacher education, and adult education. Particular attention was also paid to the question of how current symbolic mathematical systems should be improved in order to meet the particular requirements for being valuable tools for mathematical education.

As a result of the discussions, the following position statement was formulated by the group and unanimously approved:

"COMPUTER ALGEBRA: EDUCATIONAL APPLICATIONS AND IMPLICATIONS"

The second revolution in scientific computing is here. Although we expect this revolution will have far-reaching effects throughout science, the focus of this report is its impact on mathematics education. Widely available technology (in particular, many microcomputers) can support computer algebra systems, which will do the algebraic manipulation what the hand-held calculator did to arithmetic. Capabilities of these systems are symbolic as well as numeric and include, for example: expanding polynomials and collecting like terms, symbolic differentation and integration, arbitrary precision arithmetic, matrix algebra, symbolic solution of systems of agebraic an differential equations, and operations on abstract mathematical structures.

This raises immediate and long-term philosophical and practical concerns for mathematical education. How can this new power be utilized, for whom, and at what points in the curriculum? What effect will the automation of symbolic as well as numeric calculation have on mathematical thought processes?

We can distinguish three broad areas in which computer algebra will affect mathematics education.

Courses ABOUT computer algebra

Since the techniques of computer algebra are based on much traditional mathematics as well as significant recent research, courses on (as distinct from courses using) computer algebra at the graduate or advanced undergraduate level will be important in their own right. A focus on computer algebra may also provide a new framework for an exploratory approach to the teaching of much mathematics in the present tertiary curriculum.

Courses on HOW TO USE computer algebra

Utilizaton of computer algebra as a tool will have a major impact on the practice of mathematics in science and technology. Instruction on how and when to use it, its capabilites, and its limitations, will become an important part of mathematics service courses, and will also play a role in the teaching of mathematics specialists.

Pedagogical uses of computer algebra

The most profound effects will be felt in the secondary and undergraduate mathematical curricula. In contrast to the above, the details of how computer algebra will be used in these contexts are not yet clear. What IS clear, however, is that beyond providing routine training in symbolic techniques, computer algebra can dramatically enhance and enrich mathematical instruction and learning - for example:

- as a vehicle for mathematical discovery;
- by extending the range of examples which can be studied;
- as a programming environment ideally suited to mathematics;
- used to emphasize the inter-relationships between different mathematical representations;
- as an aid to preparation and checking of instructional examples;
- to promote a hierarchical approach to the development of concepts and algorithms;

Implementers of computer algebra systems need and invite suggestions on how to tailor such systems to meet educational requirements of varying student populations. Write to the Chairman of ACM SIGSAM (Special Interest Group on Symbolic and Algebraic Manipulation), 1133 Avenue of the Americas, New York 10036, USA.

A more precise idea of the capabilities of the systems and some ways in which they are already being used in education may be gained from the references, but the best method of gaining a feel for their power and likely impact is to try one: mu-MATH (available on microcomputers so the most readily accessible), MACSYMA, MAPLE, REDUCE, and others.

For references to the literature see the bibliography in Stoutemyer's paper.

1.4.2 Abstracts

Experiments With muMath In Austrian High Schools

K Aspetsberger, G Funk
Institute fur Mathematik
Johannes Kepler Universitat
A-4040 Linz
AUSTRIA

INTRODUCTION

Great progress in computer algebra (CA), both in research on the underlying algorithms and in softwares systems, has been made in the last few years. For a survey on CA see, for example, [BUCHBERGER, COLLINNS, LOOS 82]. Since symbolic manipulation is an essential component of mathematical education in today's high schools the question arises, how far CA-systems can be integrated into teaching mathematical subjects. Soon students in high schools will have easy access to CA-systems on micros. The pioneering work in this field has been done by Stoutemyer in 1980 implemeting muMATH on muSIMP [STOUTEMYER 80]. Most probably, the availability of such systems will have an influence on the curriculum that will be equally drastic to what happened when the pocket calculator replaced logarithm talbes.

CAMP: A Teaching Project in Symbolic Computation at the University of Linz

B Buchberger
Institute fur Mathematik
Johannes Kepler Universitat
A-4040 Linz
AUSTRIA

After some years of preparatory experiments, in 1982 a curriculum has been installed at the University of Linz that gives graduate students of computer science and mathematics the possibility to specialize in the field of symbolic computation (as an essential part of Computer-Aided Mathematical Problem solving = CAMP). Students of computer science are supposed to have most of the necessary software/hardware/system background and are offered a series of courses on the formal/logical/mathematical/linguistic aspects of the field. Students of mathematics are supposed to catch up with the students of computer science with respect to the software/hardware/system aspects by forcing them to pass at least those courses of computer science that are indispensible prerequisite for the field of symbolic computation. Then the students of mathematics are exposed to the series of courses on the formal/logical/mathematical/linguistic aspects of symbolic computation. The three main subareas of symbolic computation taught are: computer algebra, computational logic and automatic programming.

Intoducing Computer Algebra to Users and Students

Jacques Calmet
LIFIA/ENSIMAGE
Grenoble
FRANCE
and
Dept of Computer Science
University of Delaware
USA

INTRODUCTION

Many applications show the excellence of Computer Algebra (CA) as a computational tool in different fields of science. An unsolved problem is to evaluate it as a teaching aid in mathematics. The goal of this paper is to help to find an answer to this question.

It must be noted that some CA Systems (CAS) or programs either have been or are written for educational pruposes. Some are already commercial products and heavily advertised. This means that we are already beyond the stage of answering to the "original" question:

do we really need educational computer algebra systems? They are with us and going to stay for a long time, whether we like the idea or not.

It may be time still to ask ourselves if we want to use CAS only as a computing tool either supplementing or illustrating the skills of students, or as a means of introducing conceptual insight in some domain of mathematics.

Cayley - Teaching Group Theory by Computer

John J Cannon and Jim Richardson
Department of Pure Mathematics
University of Sydney
AUSTRALIA

THE PROGRAM

The Department of Pure Mathematics at the Unviersity of Sydney have developed a number of tutorial computer programs. Here we deal principally with Cayley, for the theory of groups, which has been used for teaching at the Unviersity of Sydney since 1975 and at the University of Melbourne since 1982. MATRIX, for linear algebra, is dicussed in a variant of the present paper. Both programs were demonstrated at ICME 5.

The programs share a common overall approach, which may be summarized as follows:

(a) Each program provides the student with a "calculator" for dealing with the mathematical objects appropriate to the area of study: matrices and vectors in MATRIX, groups in Cayley. Commoands are provided at a variety of levels, so that the calculator can grow in power as the student learns.
(b) For a given course, a file of problems is provided by the instructor. This may contain both human-language text for the student giving exercises and hints, and commands for the program to set up the required data.

The major aims of the programs are to remove the burden of arithmetic or other routine calculations from the student, thus allowing larger, more realistic, and more numerous exercises to be attempted, and to deal with common student difficulties through the hints.

Introduction to a Mathematical Laboratory

E Engeler and R Mader
Zurich
SWITZERLAND

INTRODUCTION

It is commonplace, that the use of mathematical software is in the process of influencing, if not shaping, the working style of the mathematician, physicist and engineer. Considerable effort has been put into the creation of integrated systems by various groups by:

(a) developing and collecting algorithms for various types of applications,
(b) integrating them into structured systems and
(c) providing them with facilities such as programming languages for the prospective user.

On the other hand, relatively small progress has been seen in making a telling impact on the larger scientific community with respect to wide spread use of such systems. The computer workplace for the scientist has not quite happened yet.

We feel, that this is a question foremost of education. It is imperative that we give the advanced student of the exact sciences and engineering a memeorable experience of success with using mathematical software. It is equally important that we do this without burdening the student with an inordinate investment in learning details of systems and of writing exasperatingly many lines of code.

Obviously the attainment of such an aim depends crucially on a very carefully though-out collection of representative, challenging but not forbiddingly involved, well motivated projects originating in physics, pure and applied mathematics, electrical engineering, computer sceince etc. And, of course, on the easy access to mathematical software, documentation and expert counselling.

At ETH we have been working for some years to provide and enhance such an environment. We think that we have succeeded with this pilot project. In fact, a laboratory course based on our development is now required for students in both computer science and mathematics. The present report gives and overview of the concept of our mathematical laboratory and provides some details of the projects that are presently provided for our students. Since there are more than a hundred students that take the laboratory course during a given term, we also describe some of the software support for the administratrion of the lab.

Report on the Use of Symbolic Mathematics System in Undergraduate Instruction

John Hosack, Kenneth Lane and Donald Small
Colby College
Walkerville
ME 04901
USA

Beginning in the fall of 1983 we have experimented with the use of a symbolic mathematical system in the standard undergraduate mathematics curriculum. We believe that symbolic mathematical systems are currently a useful adjunct to the standard educational techniques and will have important consequences for the future of mathematics education when symbolic systems become widely avaialable.

Self-Explanatory Symbolic Computation For Math Education

F Lichtenberger
Institute fur Mathematik
Johannes Kepler Universitat
A-4040 Linz
AUSTRIA

In this positon paper we will try to state some new ideas concerning the design of software systems for symbolic computation which are to be used in math education. Up to the late seventies computer algebra systems were running only on big mainframe computers. The muMATH-80 system was the first to be used on micorcomputers running under the CP/M operating system. Since the first hand-held computers that can run CP/M are on the market, it sems obvious that in a few years we will have computer algebra systems on cheap hand-held or even pocket computers. The use of such computational algebra systems will probably have more influence on highschool math than the apperance of electronic pocket calculators did have. If we now take a closer look at muMATH, one can see that most topics of mathematics that area relevant in high school, can be treated by the system. Some parts that had not been covered in the muMATH-80 version are covered by the muMATH-83 version, like for example vector algebra and simple differential equations. This shows the direction where things probably will go. Sooner or later all topics of high school math or even undergraduate math will be covered, at least in principle, by systems running on microcomputers. Further developments in the hardware field will accelerate this process, e.g. if the standard size for RAM storage in micros and hand-held computers incerases from now 64 KB to, say, 256 KB.

A Computer-Aided Vector-Algebra System For Use in HighSchools

Pertti Lounesto
Institute of Mathematics
Helsinki University of Technology
02150 Espoo
FINLAND

SUMMARY

A computer program has been developed to be used in conjunction with college algebra and geometry courses when introucing complex numbers and vector algebra. The input and output form of complex numbers looks like this a=1+3i. It is possible to define functions, for example f(x) = a*z + z(z-a), just writing the definiton, and evaluate their values: f(3+4i) = -7+14i. Similarly, the input-output form for vectors is the usual 5i-3j+7k or then a form applicable also in higher dimensions 5<1-3<2+7<3. The system can be used to evaluate numercially expressions containing vector formulas, like several cross products, and so students can benefit this program while doing their homework in vector algebra. The system can also be used to demonstrate subgroups, commutators, idempotents, zero-factors etc., in elementary courses on group theory and ring theory. This program is written PASCAL and it is called C LICA L.

Complexity Issues in Algebraic Algorithms: Relevant Topics for Mathematical Education

Alfonso Miola
Istituto di Analisi dei Sistemi ed Informatica
Viale Manzoni 30
00185 Roma
ITALY

The area of Symbolic Mathematics has received a wide attention in the last years, with significant results in mathematics design, systems implementaiton and development of applications. Many of these results have a direct relation to mathematical education, either as new topics to be covered in mathematical curricula or as powerful methods and effective tools for teaching and learning.

In partiuclar one can notice how the success of the researchers in Symbolic Mathematics is essentially based on a very deep understanding of the relation between algebra and computer science. In fact the proper representations of algebraic data (such as polynomials, power series, etc.) and of their algebraic sturctures (such as rings, field, etc.) have allowed very natural and simple "algorithms driven by representation". and all that matter closely links computer science to modern mathematics.

Classroom Experience With Interactive Formula Manipulation

Alexander Ollongren
Institute of Applied Mathematics and Computer Science
Leiden University
Wassenaareweg 80
2333 Al Leiden
THE NETHERLANDS

INTRODUCTION: ALGOL-68 VERSUS FORMAC-73

In a recent one-term course on adavancd concepts in programming languages the author has first dicsussed the representation of (mulitvariate) polynomials by means of structures in ALGOL-68, together with arithmetic binary - and unary operations such as polynomial addition and multiplication. Mode definitions were given and the operators were programmed. A recursive canonical form for the raepresntation of ploynomials was gvien and shown to be invariant under the operations. It was shown that ALGOL-68 is reasonably well suited for purposes of symbolic manipulation of this kind, although the amount of programming to bv done is on the whole quite substantial. As a contrast to this situation FORMAC-73 was introudced into the course, and the ease of manipulating multivariate polynomials and also truncated Poisson-series was pointed out. At the same time the resemblance between the way in which FORMAC-73 holds polynomials and the mentioned canonical form in ALGOL-68 was shown.

The Introduction of a Computer-based Tool for Solving Mathematical Problems Analytically to and Undergraduate Course

P D Pearce
Department of Information Systems Design
Kingston Polytechnic
KT1 2EE
ENGLAND

ABSTRACT

This paper suggests why it is approriate to now introduce the use of a computer based tool for solving mathematical problems analytically into a BSc degree course in Information Systems Design. Students enter this course from different mathematical backgrounds and need to be able to use mathematics itself, as a tool, to solve other problems.

A Radical Proposal For Computer Algebra in Education

David R Stoutemyer
Electrical Engineering Department
University of Hawaii
Honolulu, Hawaii 96822
USA

INTRODUCTION

I still recall the sharp disappointment felt when I realized that FORTRAN, my first programming language, was essentially arithmetic. A also recall my first mental image of a numerical analyst as someone sitting on a high stool wearing a green eyeshade and arm garters, endlessly working a desk calculator and consulting musty volumes of math tables. I ulitmately specialized in numerical analysis, but these early impressions greaty delayed my appreciation of computers and of the varied ways that they can be used in math. It is undeniably true that most students are far more intrigued and motivated by the artificial intelligence and game playing applications of computers than by the accounting and numerical scientific applications which account for most computer usage.

Why not exploit this strong preferential interest to help teach mathematics and computer skills? Computer algebra can help motivate computer enthusiasts to learn math and motivate math enthusiasts to acquire computer skills.

I propose that computer algebra is an ideal introductory and principal porgramming language for math, science, and engineering students. I propose also that the means are at hand for making it fill this role at all levels throughout our educational system. This idea may seem radical, but the supporting arguments in the following secitons ought to make the idea seem natural and timely.

Congruent Numbers -
From Elementary to Algebraic Number Theory

H G Zimmer
Fachbereich 9 Mathematik
Universitiat des Saarlandes
D-6600 Saarbrucken
FEDERAL REPUBLIC DEUTSCHLAND

Since nowadays electronic high speed computers of various sizes and types are commonly used at schools of all levels it appears to be in order to propose some appropriate mathematical problems which can be approached by means of a computer. For students this way of attacking such problems is certainly a valuable experience and should become an essential part of their scientific education.

ALGORITHMS & PROGRAMMING

2.1 Programming Languages for Mathematics Education
2.1.1 Summary

2.2 Algorithmic Approaches to Mathematics Education
Group Members:
Douglas H Crawford. Canada — Elinor McCloskey. Australia
J Lawrence. Australia — Glen Avery. Australia
Patsy B Cansey. USA — Jacques Borowczyk. France
Norm Inglis. Australia — Blagovest Sendov. Bulgaria
Chair/Reporter: Blagovest Sendov. Bulgaria

2.2.1 Summary
2.2.1.1 General Issues in the Algorithmic Approach
2.2.1.2 The Language of Algorithms in the Teaching of Mathematics
2.2.1.3 Efficiency of an Algorithm
2.2.1.4 Problem Solving With and Without Algorithms
2.2.1.5 Known Experiments in Development Algorithmic Thinking

2.3 Technology and Cognitive Development
Group Members:
G Ball. Australia — A Ballard. Australia
M Barnes. Australia — M Brown. Australia
A Elliot. Australia — W Ransley. Australia
D Lavallade. France — A K Das. India
D K Sinha. India — S P Yadav. India
H Hirabayashi. Japan — I Nishtiani. Japan
H Okamori. Japan — K Yokochi. Japan
J Sauer. New Zealand — R Selden. Papua New Guinea
P Laridon. South Africa — J Vermeulen. South Africa
P Saengcharoenrat. Thailand — J Becker. USA
P Campbell. USA
R & R Hutter. USA — D Kressen. USA
D Resek. USA — B Travis. USA
R Wenger. USA — G Wheatley. USA
E Cohors-Fresenborg. West Germany — W L Fischer. West Germany
K Haussmann. West Germany
W Lorbeer. West Germany — M Reiss. West Germany
I Wachsmuth. West Germany
Chair/Reporter: J Becker. USA
I Wachsmuth. West Germany

2.3.1 Introduction
2.3.2 The Use of Technology to Further the Cognitive Development of Learners of Mathematics
2.3.2.1 The Computer as an Aid to Develop Mathematical Concepts
2.3.2.2 Cognitive Development Through Active Engagement in Programming
2.3.2.3 The Computer as a Tool for Problem Solving and Exploration
2.3.2.4 Developing Imagery and "Big Ideas" Through Computers
2.3.3 The Use of Technology to Achieve a Better Understanding of the Processes Involved in Children's Development of Cognitive Structures Relevant to Mathematics

2.3.3.1 Investigating Cognitive Development in the Context of Computers
2.3.3.2 The Role of Artificial Intelligence Research and Technology for Understanding and Furthering Cognitive Development
2.3.4 Possible Disadvantages Connected with the Use of Technology
2.3.5 Summary
2.3.6 References

2 Algorithms & Programming

2.1 Programming Languages for Mathematics Education

Group Members:
Douglas H Crawford. Canada
J Lawrence. Australia
Patsy B Cansey. USA
Norm Inglis. Australia
Elinor McCloskey. Australia
Glen Avery. Australia
Jacques Borowczyk. France
Blagovest Sendov. Bulgaria

Chair/Reporter: Blagovest Sendov. Bulgaria

2.1.1 Summary

In examining the relationship between programming languages and mathematics education, it is important to consider the relative emphasis which should be placed on computer science and on mathematics even though there are strong areas of overlap between the two disciplines. From the perspective of the mathematics educator it seems clear that computer languages should be seen essentially as tools rather than as objects of study in their own right. A characteristic of the technology which is especially important in this context is the opportunity it offers for the dynamic visual representation of mathematical concepts.

The impact of computer technology on mathematics education is complex and defies simplistic description or recommendations. In the recent past, at least in the so-called developed world, there has been very rapid growth in public accessibility to computers mainly because of the advent of the microcomputer. This growth, which shows little sign of abating, has been paralleled by that of an active software industry which has produced numerous versions of 'classical' languages such as BASIC, Logo, Pascal, Pilot, Fortran and APL. More recently, other languages such as Prolog and Smalltalk, have been the subject of much discussion. As has historically been true with other educational innovations, there would seem to be considerable difficulties associated with the transfer of this technology to the developing world.

Onc can identify two principal roles that a programming language could play in mathematics education:

(1) as an arena for student-directed exploration, and
(2) as a medium for the development of teacher or curriculum support materials.

Languages such as APL and Logo appear to be particularly well studied for 'doing' mathematics, permitting the student to engage in self-initiated and open-ended exploration. APL was originally developed as an alternative to conventional mathematical notation and has been recommended by some for more common use even if one does not have access to a computer. Logo was constructed with young learners very much in mind and is particularly well-suited for the elementary grades although it has, in its full implementations, the capability for use at much higher levels. Examples are beginning to appear which illustrate a different pedagogical approach where one begins with a rich problem area or mircoworld and uses this as an entry point to relevant mathematical topics.

2.2 Algorithmic Approaches to Mathematics Education

Group Members:

Douglas H Crawford. Canada	Elinor McCloskey. Australia
J Lawrence. Australia	Glen Avery. Australia
Patsy B Cansey. USA	Jacques Borowczyk. France
Norm Inglis. Australia	Blagovest Sendov. Bulgaria

Chair/Reporter: Blagovest Sendov. Bulgaria

2.2.1 Summary

2.2.1.1 General Issues in the Algorithmic Approach

- the advent of the computer forces us to reassess and reinterpret the idea of the "algorithm", not only in mathematics but also right across the curriculum.
- algorithmic approach goes well beyond the definition of ALGORITHM - viz. a finite sequence of instructions which, when carried out, will solve in a finite number of steps all problems in the category for which the algorithm is designed.
- algorithmic approach is limited to finite systems. Infinite systems do not lend themselves to algorithmic treatment unless first approximated by finite systems.
- some algorithms, e.g. those in elementary school for addition, subtraction, multiplication and division, do have to be learned even though they are "rote". These are the mathematical equivalent of "walking", and are a necessary preliminary to any mathematical development.
- as teachers, we have not always clarified the concepts in our own minds nor have we communicated the concepts to students appropriate to their level.
- a good algorithm does not just get an answer but enhances understanding of a concept. Developing one requires a strong understanding of the process.
- emphasis on algorithms is frequently misunderstood and arguments such as "Why bother when calculators have them built in already?", and "students get answers without understanding what they are doing", still persist.

2.2.1.2 The Language of Algorithms in the Teaching of Mathematics

- the characteristics of algorithms should reflect the age group. ability, pedagogical purpose being served. There is HOW?, WHEN?, WHERE?, WHY? with every use.
- algorithmic approach requires good symbolism. Even the use of "pidgin" BASIC was seen as a good vehicle for the synthetic division algorithm.

2.2.1.3 Efficiency of an Algorithm

- what is the importance of efficiency? (e.g., long division vs. synthetic division in division of polynominals.)
- non-efficiency may be didactically useful.
- an algorithm to help learning is not necessarily the same as an algorithm for expediency.
- for a very weak child, any algorithm it can use is a good one, whether understood or not.

2.2.1.4 Problem Solving With and Without Algorithms

- there is an need to distinguish between what has to be learned by heart and what has to be arrived at by logical processes.
- algorithms must not eliminate the need for students to think.
- some algorithms are better "programmed into" children, e.g. area of rectangle (primary), solutions of quadratic (secondary), in order to free them for more advanced work.
- other problems have to be solved without aid of an algorithm, e.g., "work" problems, which require creative thought.
- a student who learned and understood an algorithm will "recover" it much quicker after a period of non-use than one who only used it without understanding earlier.
- not knowing an algorithms. knowing that it is not possible to find an algorithm ... only for sophisticated students.
- in revision, enrichment etc., solution by several different algorithms may by useful but not in early stages of dealing with a problem.
- there is a place for students to create their own algorithms.

2.2.1.5 Known Experiments in Development Algorithmic Thinking

West Germany - actually introducing courses in the theory of algorithms. More information available?

Bulgaria - Professor B Sendov, Vice-President of the Bulagarian Academy of Sciences, outlined an experiment in his country in which great stress is placed on skills in symbolic expression and algorithmic thinking at early primary age.

2.3 Technology and Cognitive Development

Group Members:
G Ball. Australia
M Barnes. Australia
A Elliot. Australia
D Lavallade. France
D K Sinha. India
H Hirabayashi. Japan
H Okamori. Japan
J Sauer. New Zealand
P Laridon. South Africa
P Sa engcharoenrat. Thailand
R & R Hutter. USA
D Resek. USA
R Wenger. USA
E Cohors-Fresenborg. West Germany
W Lorbeer. West Germany
I Wachsmuth. West Germany
A Ballard. Australia
M Brown. Australia
W Ransley. Australia
A K Das. India
S P Yadav. India
I Nishtiani. Japan
K Yokochi. Japan
R Selden. Papua New Guinea
J Vermeulen. South Africa
J Becker. USA
P Campbell. USA
D Kressen. USA
B Travis. USA
G Wheatley. USA
W L Fischer. West Germany
K Haussmann. West Germany
M Reiss. West Germany

Chair/Reporter: J Becker. USA
I Wachsmuth. West Germany

2.3.1 Introduction

This article reports on information disseminated and discussions and recommendations elaborated in the theme group on "Technology and Cognitive Development (Topic 2.3 within the Technology theme) at ICME 5 in Adelaide, Australia. The group focused on three questions around which the report is organized:

(a) The use of technology to further the cognitive development of learners of mathematics;
(b) The use of technology to achieve a better understanding of the processes involved in children's development of cognitive structures relevant to mathematics;
(c) Possible disadvantages connected with the use of technology.

It appeared that "technology" was nearly exclusively thought of as (micro)computers. "Cognitive development" was taken to be a long-term process of change in children's mathematical intellectual behaviour. Several group members listed in the references of this article gave introductory papers or catalyst statements which are amalgamated into this article together with information and opinions that were contributed in the discussions. A questionnaire completed by the group members on the last of the four working days provided further background information for the report. Student ages reflected in participants'contributions and discussions range from early elementary through the tertiary level.

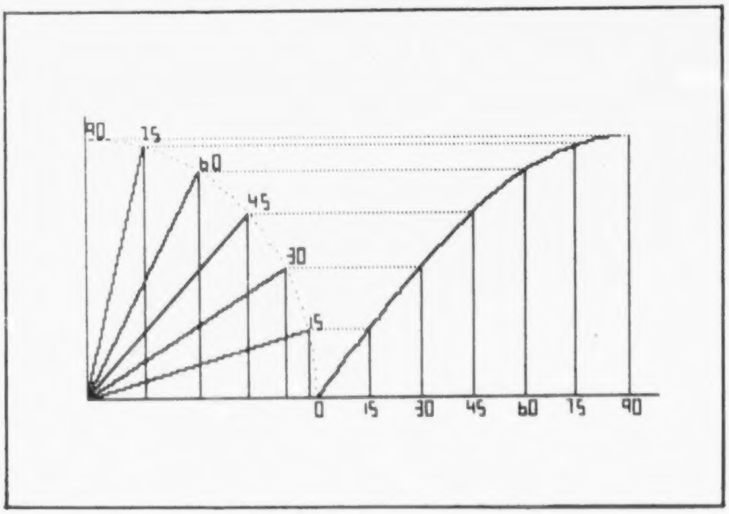
2.3.2 The Use of Technology to Further the Cognitive Development of Learners of Mathematics

2.3.2.1 The Computer as an Aid to Develop Mathematical Concepts

We begin by presenting examples of work in which micro-computers are used to support concept development in content in the tradtional mathematics curriculum. Ruth S Hutter (Riverdale Country School, Bronx, USA) reported that at present most computer programs used in mathematics instruction fall into two categories, commonly called drill and practice and so-called teaching programs. The teaching programs furnish students with explanatory text and graphics in addition to having them answering questions and scoring their responses. Hutter viewed this procedure essentially as another form of drill which, though necessary, should be only a small part of the teaching process.

In contrast, the purpose of Huters's presentation was to emphasize the invaluable role which the computer can play as a visual aid for clarifying mathematical concepts and facilitating understanding of conceptually difficult topics. While blackboard and textbook illustrations should remain primary tools for classroom instruction, these media are less helpful when a succession of drawings is required to present concepts such as the limiting processes of calculus or the changing images in transformations. Here, in contrast to films, the computer offers unique flexibility in that programs can be stopped at any time and parameters in equations can be varied at the will of the teacher or the students. While it has been said that a picture is worth a thousand words. Hutter felt inclined to say that computer graphs are worth a thousand pictures.

Ruth and Rudolf Hutter have developed a series of programs for traditional mathematical content that are devised for use by an instructor. The programs are not geared to any particular textbook but, rather, provide sufficient freedom for the teacher to order and style presentations. The programs include topics for calculus, precalculus, trigonometry, and intermediate algebra. Figure 1 illustrates how the sine curve is derived from



the ratios of a side and hypotenuse in a right triangle with progessively enlarged angles in the unit circle (dynamically generated by the computer). After viewing the programs, group members had the impression that the dynamics of the images produced on the computer screen could potentially lead to good imagery supporting understanding.

Having used the programs in the classroom, Ruth Hutter reported that the attention and involvement of students of various abilities seems greatly enhanced. She also observed continuous mental involvement of students and a better retention of the topics discussed, as well as a facilitation of the use of mathematics in applications. Further work on such classroom use of the computer should concern the frequency of use, time limits (physical considerations like eye strain), required changes in textbooks, and production of new materials made necessary as a consequence of this use of new technology.

Kiyoshi Yokochi (Yamanashi University, Kofu, Japan) has developed microcomputer software concerned with the teaching of number for Japanese preschools and had also carried out several classroom experiments in order to locate critical parts of instruction where using such software could be beneficial. He identified a succession of a three steps by which number teaching proceeds:

(1) An introduction to the content in which children make good use of real objects;

(2) the grasp of number concepts and rules which require that children make good use of the teacher's explanation and workbooks;
(3) an application of what has been learned in real-life situations.

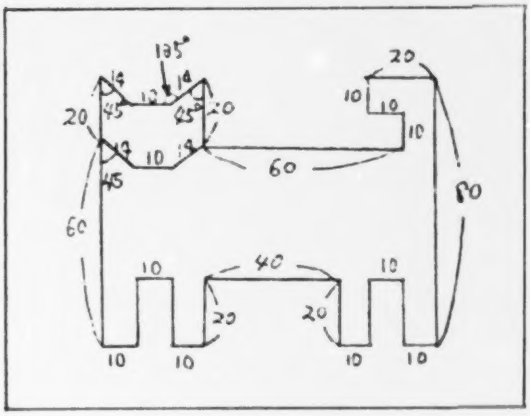
Yokochi pointed out the importance of the second step which is not reflected in the reality of many schools. Often emphasis is placed on the first and the third steps while in those schools placing emphasis on the second step instruction tends to be "dry as dust". Here Yokochi sees the point at which the microcomputer and software can be used that not only fascinates children but also facilitates development of number concepts and rules in children but also facilitates development of number concepts and rules in children.

The sequence of software lessons is ordered according to the development of children's cognition of number. The story line of lessons develops situations that are meaningful to the child's reality (e.g., playing soccer) where the "hero" is the child operating the computer. Through answering questions posed at many points in the lessons, the child is expected to grasp important aspects of the number concepts and to learn the rules to be mastered. Random sequences of questions, as well as music and graphics, are used to encourage the child to work through the lessons.

In most Japanese preschools, the number of students in a class is about 30 with at most 15 microcomputers available per classroom (sometimes fewer). The microcomputers are used in either an individualized or an all-students-together learning fashion. For an efficient use of the software Yokochi recommends that the software-assisted lessons be integrated into the usual course of lessons such that the computer is used about once a week to traverse the "grasp" step between the "introduction" and "application" steps. Weekly repetition of computer lessons are continued until all questions are answered correctly in the case of individualized learning, or until 90% of the children give correct answers to all questions in the case of classroom use of the software.

In summarizing the effects of this software on children, Yokochi noted that the children appear to be so absorbed by the lessons's stories that they don't want to miss class. Further, every child masters the content which, in general, is not expected from the usual approach to instruction. (For related literature, see Yokochi, 1983, and Yokochi & Machida, 1982).

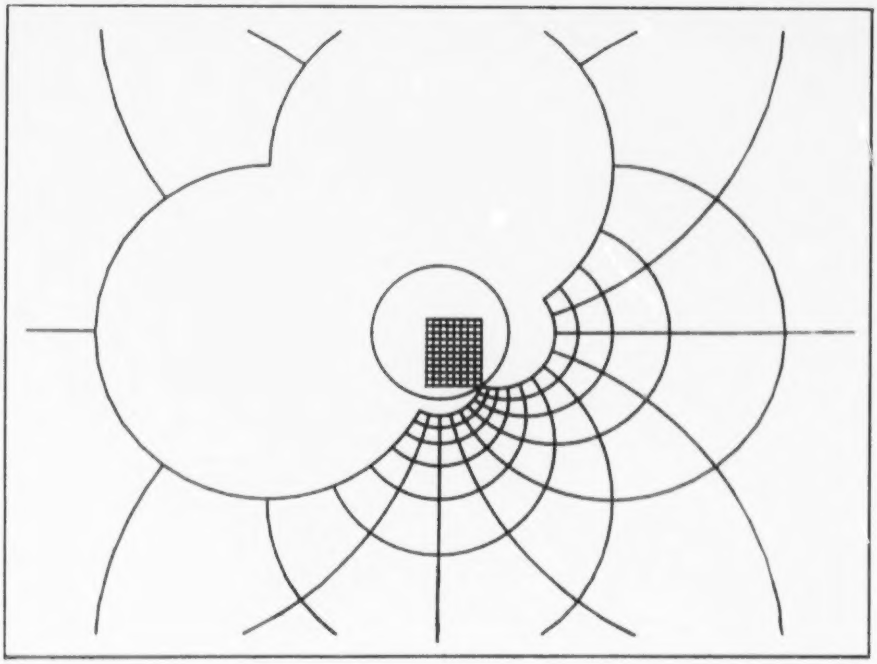
Co-workers of Yokochi, Hirokazu Okamori and Isumi Nishitani have used Logo with a small number of fifth grade elementary school children. The project's aim was to develop children's comprehension of the similarity of figures, to have them learn a simple technique of measuring heights of well-known towers in Japan, to use the computer to calculate heights from reduced drawings produced on the screen, and to write Logo programs for magnifying or contracting figures. Problems children were asked to solve included writing Logo procedures that output an animal (shown in Figure 2) and one in which variables were used to output an "approaching" dog (supposedly in a succession of progressively magnified images). Okamori and Nishitani noted that while the children were not good at



mathematics, by working with the computer they came to understand the property of similarity and the foundation of Logo programming. The computer was effective in motivating children and in helping to develop mathematical ideas in children.

Walter L Fischer (University of Erlangen-Nürnberg, West Germany) noted that the introduction of computers into mathematics instruction has at least two dangers: on the one hand computers could be used as mere calculators and, on the other, they could be used as highstyled playthings by teachers enraptured with programming. Using computers in such ways would occur on levels too high or to low, respectively, with respect to students' capacities and realistic educational aims. Fischer emphasized that for a systematic incorporation of computers into the curriculum, well-defined educational aims need to be identified, the achievement of which involve the use of computers. Whether current curricula can be modified to incorporate use of computers or whether the necessities of the computer age require curricula of a completely different structure is a serious question which mathematics educators need to address.

Following the first line, Fischer has explored how computers can be incorporated into geometry instruction. The systems consisting of computer and plotter (or screen) can be interpreted as a universal drawing device, the use of which is constrained by a set of propositions similar to those imposed on the use of compass and ruler. In pursuing the "Erlanger Program" of transformational geometry proposed by Felix Klein, Fischer has found that the superior graphics capabilities of the microcomputer can help to overcome one of the shortcomings of this approach, namely, that instruction is usually confined to the group of Eculidean motions because of the complexity of drawings. As an example, Figure 3 shows a transformation by reciprocal radii produced by the computer, the genesis of which can be explained to students while the plotting is in progress.



Fischer concluded by saying that in lower grades computers can be used as a medium to impart knowledge of geometrical facts to students, whereas higher grades students should be able to conceptualize geometry programs for the computer on their own. The acquisition of programming skills will then be connected with educational objectives in geometry, and visa versa. In this stage, the computer can become an instructional tool as well as a medium to develop abilities and skills.

The last two examples presented in this section report on activities in developing countries. Dilip K Sinha (Jadavpur University, Calcutta, India) illuminated some aspects of ccomputer education from the perspective of a developing country. He reported that, besides the difficulties computer education faces in India due to the developing phase of industrialization, which does not yet permit wide use of computers, the possible erosion of computational abilities is a purported weakness frequently expressed in this country of rich mathematical heritage. SINHA sought a general discussion of the point that understanding in mathematics can be reinforced through activities using the computer. He stated that no meaningful pedogogical program can bring about fruitful uses of innovative technology unless the issue of cognitive development is studied and related to a setting of mathematical education with an integrated computer component. Taking the instructional aim of achieving mathematical understanding and the consequent elimination of rote learning as prime imperatives, one has to look for appropriate uses of the computer towards these ends.

Sinha and his co-worker G Bajani suggest that these aims can be achieved, in an ample measure, through studies in computer topics which are interwoven with mathematical topics. Seeking algorithmic approaches to strictly mathematical studies could add new perspective to the cognitive development of mathematics learners. A pilot project testing a computer education programme in some selected schools which was recently initiated in India as expected to provide a substantial input to future investigations in this direction. As a fist step, a workshop held by the National Council of Educational Research and Training (1984) in India has presented a report on Computer Literacy and Studies in Schools (CLASS").

Porama Saengcharoenrat (Prince of Songkla University, Thailand) is involved in developing programs on Computer Based Learning in her country. One way in which she sees that computers can help university students learn mathematics is described as follows. Classes at the universities in Thailand tend to be overcrowded, so that teachers have on the average rather little time for the individual student. Furthermore, it is an unfortunate characteristic of Thai students that they usually lack courage to ask questions and to try to clarify unclear points. In both these respects the use of the computer appears to be helpful. When students study at their own terminals, they can proceed at their own pace and can go through the lessons as many times and as carefully as they like in order to understand the subject more completely. Difficulties arise from the fact that only teaching packages written in Thai language will facilitate a use of the computer by a wide group of people. Since Thai script has a very complex notation, many technical problems need to be solved before the situation can be improved (see also Saengcharoenrat, 1984).

2.3.2.2 Cognitive Development Through Active Engagement in Programming

The examples so far are concerned mainly with ways of using computers that were pre-programmed to provide a learner with demonstrations or instructional material. A very different approach, which is still somewhat controversial regarding what kind of mathematics learning is involved, is the "environmental approach" typified by Seymour Papert's Logo Laboratory. The observation is sometimes heard that overusing the computer in certain ways might in a sense lead to "programming" the child. A quotation from Papert's (1980) book Mindstorms serves as a response when he notes: "The child programs the computer. And in teaching the computer how to think, children embark on an exploration about how they themselves think. The experience can be heady: Thinking about thinking turns the child into a epistemologist, as experience not even shared by most adults."

Another criticism sometimes heard is that of a mechanical kind of thinking being produced in children as a result of working with the computer. Says Papert (in Mindstorms): "by deliberately learning to imitate mechanical thinking, the learner becomes able to articulate what mechanical thinking is and what it is not. The exercise can lead to greater confidence about the ability to choose a cognitive style that suits the problem. Analysis of 'mechanical thinking' and how it is different from other kinds and practice with problem analysis can result in a new degree of intellectual sophistication. ... Instead of inducing mechanical thinking, contact with computers could turn out to be the best conceivable antidote to it. ... Through these experiences these children would be serving their apprenticeships as epistemologists, that is to say learning to think particularly about thinking."

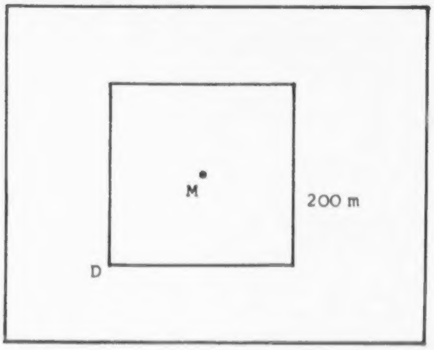
Carrying this aspect further, it seems a sensible role for computers in mathematics education to enable children to observe, and become reflective about, their own actions (carried out by a computer they have programmed). Thus, we wonder whether programming a computer may stimulate thinking experiences that are beneficial to the learning of mathematics and, especially, problem solving. Children can observe and become reflective about their own thinking and thereupon use it more effectively (technology as a "thinking aid"). The dialogue style of interaction with some more advanced systems might even enhance that feature, as was reported by several participants.

2.3.2.3 The Computer as a Tool for Problem Solving and Exploration

Some presentations emphasized possible uses of computers as a tool in problem solving in mathematics. Jerry Becker (Southern Illinois University at Carbondale, USA) reported on such use of the microcomputer in the secondary mathematics classroom. In a project he co-directed (Becker & Pedersen, 1983, June) students were taught the rudiments of programming in Apple Pascal (Carmony, McGlinn, Millman, & Becker, 1984) and then attention was turned to solving problems. Problems posed were those which could be solved with the use of a microcomputer as a tool in conjunction with application of various problem solving heuristics to which students were exposed earlier (see Schoenfeld, 1983, and Burton, 1979. Two examples follow:

(a) Note that 153 has the property that $1^3 + 5^3 + 3^3 = 153$. What other 3-digit numbers have this property? 4-digit numbers? 5-digit numbers? etc.

(b) A vicious dog (D) is at the southwest corner of a square garden 200m on a side (see Figure 4). A man (M) is in the centre of the garden. The dog will not attack the man in the garden, nor will it attack the man once he has left the garden. But the dog will run along the wall and try to bite the man as he crosses the wall. If the dog runs   times as fast as the man, is there a point(s) to which the man can run to escape the dog without being bitten?



The microcomputer proved to be useful in the solution of these and many other such problems and may contribute to cognitive development in several respects: students tend to become *reflective* about their thinking and actions - they must organize their thinking and then take action in using the microcomputer as a tool; students' use of the microcomputer to explore and exhaust all possible cases may serve as an additional heuristic and students may evolve a new perspective on mathematical problem solving as a result of these experiences. Algorithmic thinking may also become part of the students mental processes, and, thus, the development of problem solving skills may be enhanced by use of the microcomputer.

In a separate part of the project, Papett's Turtlegraphics were used with elementary level students. Here it was found that, almost without exception, students reported that they were *always thinking* while interacting with the microcomputer. Related to cognitive development were the following observations: students thought through a sequence of actions, related well to visualization of their actions, and learned to see equivalences - several programming steps can be done in one step.

It was observed during discussion that we may not know exactly *how* the microcomputer contributes toward cognitive development, but there was general consensus that it may have beneift and that further efforts to study this issue are justified.

Grayson H Wheatley (Purdue University, USA) has also worked on using the computer as a problem solving tool. Wheatley starts from the observation that beliefs and expectations play a major role in mathematics problem solving. If, as it has been argued by educators favouring the constructivist approach of learning, students must construct knowledge for themselves, then using the computer as a problem solving tool is a valuable experience. By organizing ideas around a problem, students can build personal schemas that can be readily applied to a wide variety of new situations. A major strength of solving problems through programming is that the user is in control. This is in sharp contrast to other modes in which the user responds to questions displayed on the screen or is "programmed" though a set of tasks. Furthermore, a programming language can be learned effectively in the process of solving mathematical problems. Finally, computer problem solving encourages the student to develop a "relational" view of mathematics that draws understanding from observations of the cause effect dependency between the different concepts and rules involved. in constructing the solution.

The second point emphasized by Wheatley is that computing devices, whether computers or calculators, can be important tools for exploration. Using a computer graphing program to explore the nature of functions can place the emphasis on mathematics rather than computation. Wheatley reported findings which show that students make more exploratory moves in problem solving with the use of calculators. In a study, he found that in problem solving 11-year-olds performed more exploratory computations, used a wider variety of heuristics, and completed the same number of problems in half the time with better performance. When students expect to get stuck on a problem and expect to explore several approaches, they are more successful than students who expect to apply a rule.

Studies at Purdue University have also shown that bright eleven and twelve year olds became quite adept at writing programs (in Basic) to solve mathematics problems like "Find three positive integers such that their sum is 43 and the sum of their cubes is 17299." A twelve-year-old with computer problem solving experience, when confronted with an algebra word problem outside the context of using computers, immediately sketched a computer program based on exhaustive search which he could subsequently refine to obtain the correct solution. While this boy had been unable at age 11 to set up an equation and solve it in the traditional way, the new tool now enabled him to handle a problem reasonably challenging for 15-year-olds. Similar observations with other students from the same class substantiate the impression that the heuristics used in computer problem solving, of which exhaustive search is one, are quite powerful in that they apply to a broad range of problems.

Examined in the light of computer availability, Wheatley put forward the argument that the heavy emphasis which traditional courses place on learning procedures for computing quotients and roots, as well as solving systems of equations, for example, may no longer be appropriate. On the other hand, students relying too heavily on computer-related strategies may not develop the relationships which are acquired by looking for patterns and attempting to build a representation that characterizes the structure of the problem. While it has been shown that solving problems by constructing computer programs is an excellent setting for enhancing problem solving abilities, Wheatley also calls for investigation of undesirable side effects like that of overusing computing facilities which might impair the development of pattern finding strategies.

2.3.2.4 Developing Imagery and "Big Ideas" Through Computers

It has been noted that imagery plays an important role in high-achieving mathematics students while less able students may experience more difficulty in understanding mathematics which requires images, and this may be especially true when it comes to "abstract" mathematical concepts and relationships. In this connection, Robert B Davis (University of Illinois, Urbana, USA) made reference to two different views of teaching: (a) one that views students learning by following explicit instruction, and (b one that views students "building up" mental representations in their mind while interacting with an environment. He made the point that a large amount of computer-assisted instruction treats knowledge as if it were co-extensive with vocabulary drill which is not entirely surprising since this approach has been a weakness of much education in pre-computer forms. By treating "Knowledge" as nothing more than "knowledge about words", one confuses, as Davis put it, encyclopedias and dictionaries - the former being properly a book of knowledge, the latter a book of words.

Davis emphasized the importance in learning which much implicit knowledge has that is not stored in one's mind in the form of actual language statements. A great deal of such knowledge is derived form experience in an individual's dealing with aspects of the knowledge. Technology can offer ways of teaching children without using any words. An example of such teaching material aimed at building up a reasonable notion of fraction size is a computer game called DARTS. In this game, a vertical number line appears on the screen with indicated end marks and balloons attached at a random positions. By typing in a fraction, dart can be made to fly across the "pop" a balloon if the fraction is a close-enough estimate.

Davis continued by mentioning the role which technology could play as mediator in building up "big ideas" in a learner's mind; e.g., the important concept of covergence of an infinite sequence. He referred to the notion of "frame" structures that organize such knowledge in a way that the whole is more than the sum of its parts (cf. Davis, 1984). The understanding derived from such big ideas is often metaphorical, not logical in that one uses ideas already developed as a way to approach new ones which sometimes require that slight modifications or adaptations be made. As an example from physics, Davis mentioned how Rutherford scatterings became understandable thanks to the "big idea" of the solar system; a revised model of the solar system was further used to create an "atomic structure" model (cf. Gentner & Stevens, 1983; Davis, op.cit.).

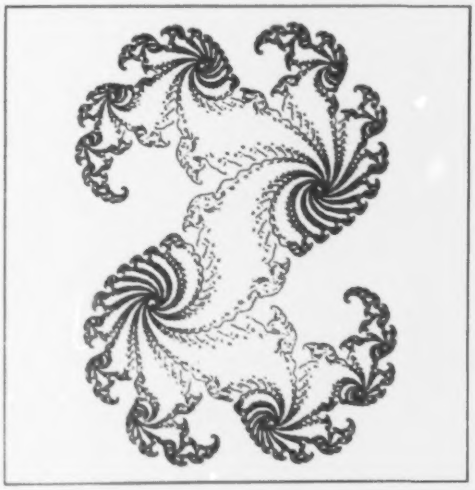
Gerald Golding (1983) has presented some thoughts which reinforce these ideas from which we quote a paragraph here:

> "It is an interesting conjecture that 'genius' in mathematics or physics has to do mainly with the imagistic level. It seems unlikely that the unusually great mathematician achieves that greatness merely by superior encoding of verbal problem statements in formal symbols, or by more efficient processing of formal notations, or by much better and more sophisticated heuristic plans. It is more plausible that the innovator has somehow succeeded in constructing a superior imagistic (non-verbal) representation, within which mathematical relationships inacessible to others can be 'seen'. The challenge of mathematics for such a person is to develop appropriate notations within which such relationships can be precisely described and axiomatized, so that theorems can be proven."

We conclude that it is a challenge for us to use technology to develop appropriate methods that can support imagistic thinking of "abstract" mathematical situations and relationships among all students. The graphic capabilities of microcomputers are a possible means to achieve visualization. Examples shown and discussed in the working group emphasized the dynamical aspect of computer graphics, with the idea of creating moving images preparatory to the development of important abstract concepts ("big ideas").

Ipke Wachsmuth (University of Osnabrück, West Germany) provided two examples from outside of the theme group that were discussed to outline further the far-reaching possibilities proved by professional computer graphics. Thomas Banchoff (1978) has become known for his computer animations of the geometry of surfaces in 3- and 4-space. At Brown University, he has used a high-speed graphics computer to display on the screen a sequence of images which the viewer readily interprets as the projections of an object rotating in 3-dimensional space. By turning dials, one can investigate a curve or surface by having it rotate about different axes and stopping it at especially interesting positions. One can "fly inside" the object to focus on some local behavior or proceed to examine some specific singularity by deforming the object thorugh a one-parameter family of curves or surfaces.

The other example concerns computer graphics from the theory of dynamical systems. At the University of Bremen, West Germany, H O Peitgen and P H Richter(1984) have graphics computers produce "Gedanken maps" to visualize characteristical features of mathematical feedback systems, as in, for example, the Newton iteration procedure for certain complex equations. Pictures never seen before illuminate the situation in regions of dramatic change in the system's behavior of convergence, like the "Julia sets" separating the domains of competing attractors, or like the "Mandelbrot sets". Figure 5 shows a very modest graphic (reproducable in black-and-white) of a Julia set; other graphics are very colorful and richer in terms of coded information. A case has been reported where a result obtained theoretically was shown to be false by looking at such a a map.



A kind of "experimental mathematics" has evolved from these approaches which may be progressively more important for an understanding and mathematization of complex processes. A group, working around W Metzler at the University of Kassel, West Germany, is exploring visualizing mathematics and has started to explore possible uses of such technology, also within the reach of microcomputers, for didactical purposes (see Metzler, Beau, & Uberla, 1985).

2.3.3 The Use of Technology to Achieve a Better Understanding of the Processes Involved in Children's Development of Cognitive Structures Relevant to Mathematics

2.3.3.1 Investigating Cognitive Development in the Context of Computers

Elmar Cohors-Fresenborg (University of Onsnabrück, West Germany) reported on research aimed at introducing a fundamental understanding of computers to children (age 10-13) which is based on elementary actions, and, secondly, on cognitive strategies observable in children's construction and analysis of computer programs. Concerning the first topic, he described a teaching project introducing the concept of functions on the basis of algorithms (Cohors-Fresenborg, Griep, & Schwank, 1982). A fundamental hypothesis of the project is that the central problem in programming is to organize a sequence of actions to be executed by the computer. In this approach the set of whole numbers is used as an initial data structure where the elementary operations performed on it are counting forward and backwards. The control structures are comprised by concatenation of partial algorithms and the iteration of algorithms (while a storage remains non-empty).

The approach includes three different ways of representing an algorithm:

(1) The whole numbers are embodied by a heap of sticks, then the elementary operations in an algorithm are adding or taking away a single stick;
(2) an algorithm is represented by a so-called computing network which constitutes a functionable flow chart and is constructed with a special manipulative called "Dynamical Mazes";
(3) an algorithm is represented as a program for a special model computer, called "Registermachine", which is implemented on a usual microcomputer.

Classroom experiments have proved that, by this non-verbal approach to algorithms, 13-year-olds easily get a fundamental understanding of constructing and analyzing algorithms and a first introduction to a programming language which, from the mathematical point of view, has the complexity of Pascal.

A second part of the project is devoted to the role which the three different ways of representing algorithmic concepts play in concept formation of 13-year-olds (Cohors-Fresenborg, in press) and to different cognitive strategies exhibited by children when they construct or analyze algorithms (Cohors-Fresenborg & Kaune, 1984). A number of case studies have shown that the above-mentioned three ways of representation do not form a hierarchy. Some children prefer to begin at the level of handling sticks, while others prefer to first construct a computing network.

The case studies have also shown that there exists at least two different cognitive strategies in constructing algorithms for the Registermachine. One strategy may be described as follows: children reflect on how they would organize the acting of the Registermachine sequentially. Questionned to describe what a program does, they use words indicating that they are imagining a Registermachine acting step-by-step. But there are also children that begin inventing a computer program by structuring the given problem beforehand, using concepts they know from earlier problem solving experiences. In the first case one could speak of a sequential and in the second case of a conceptual strategy.

Patricia F Campbell (University of Maryland, USA) has focused her recent studies on the young child in a microcomputer environment.

She mentioned that there was concern in the USA that the microcomputer could destroy the environment of preschool and primary school. In particular, would microcomputers discourage or distort the social and emotional development of young children, replacing active manipulation and experiential learning? On the other hand, the microcomputer has the potential of providing the young child with an environment replete with symbolic and spatial experiences. Campbell and her colleagues at the University of Maryland are exploring how young children interact with a microcomputer and how their level of development may influence their interaction.

In a first study (Fein, Campbell, & Schwartz, 1984) computer graphics were regarded as early symbolization. At the Center for Young Children (CYC) at the University of Maryland, four-year-olds had access to microcomputers four days a week during their free choice period. Initially the children viewed the microcomputer as a toy, as they learned to use a commerically-produced graphic art packages. However, after first exposures had produced an understanding of the cause and effect relationship between the controls and the screen display, manipulations became more functional in nature as the children learned to control the display and make the cursor move on the screen. The play became constructive rather than functional as the children drew images on the screen in order to produce a product. Sometimes children had a final product in mind prior to the initiation of the drawing. This type of play expanded as the children began to attach a symbolic meaning to a dynamic image on the monitor screen, a meaning which they could easily change in their "pretend" world.

Observations of dramatic play at he microcomputer also revealed instances of collective symbolism as pairs of children collaborated to define, share and modify the meanings of created objects and scenes in their make-believe-world on the monitor screen. Rather than promoting isolation, the microcomputer led to discussion and social interaction (Wright & Samaras, in press). One key difference between a computer graphics environment and that available via crayons and paper was dynamic control. With the microcomputer, misplaced lines could be quickly erased, colors could be changed, and images were easily modified. For a four-year-old experiencing the frustration of inefficient motor control, the potential of the illusion of movement and the option of change made this an art project that was more interesting to play with and to share.

A second study which Campbell reported explored pre-schoolers' dealing with Logo, from a syntactic and spatial perspective (Campbell, Fein, Scholnick, Frank, & Schwartz, 1985). While it has been argued that learning to program a computer will promote cognitive benefits such as facility with problem solving heuristics and the development of procedural thinking, some researchers have questioned these conclusions, as Campbell points out. At the most primitive level, programming requires a comprehension of the syntactic potential of the programming language. Although proper studies have documented that children can program in the Logo language, there is little research on what the constructs of programming in Logo are or how they are mastered. The study conducted recently at the CYC examined five-year-olds' emerging in competence with the syntax and semantics of the Logo language (using a simplified version, "Instant Logo") and, further, their ability to take the perspective of the triangular cursor (turtle) and to reorient accordingly in a two-dimensional space. The project did not view the learning of Logo as the development of programming skills which would be beneficial for transfer to other areas; rather, learning Logo was viewed as a positive goal in itself.

A preliminary data analysis reveals "findings" that the reflective control of two successive commands involving distance and direction emerged gradually for the children. They had a definite bias for the forward command for distance and the right command for direction, and utilization of the four graphic comands seemed to occur more effciently when the cursor was in the HOME position (center screen, heading up). Many of the children exhibited revision skills that seemed to expand into monitoring skills as they began to interpret and compensate for the subsequent movement of the cursor. Some children seemed to be able to estimate a unit of measure, primarily for distance, and they were able to apply this as they moved the cursor to a specified location or reproduced a given line segment.

A hypothesis offered by Campbell describes the process of children's learning of Logo in the following manner: The meaning of the four graphic commands is mastered readily. Awareness of the need to order direction commands prior to distance commands emerges gradually, producing a primitive system of commands based on isolated movement. Recognition of the reversible relationship between left/right commands and forward/back commands permits revision of the prior entry. When this level is reached, the system of Logo commands is not only based on isolated movement, but it is also a compensatory system. Eventually there is an understanding that a single command or move may be embedded in a whole series of commands in order to produce a pre-determined image or a component of a figure.

2.3.3.2 The Role of Artificial Intelligence Research and Technology for Understanding and Furthering Cognitive Development

In what ways can technology be used to achieve a better understanding of the processes involved in children's cognitive development with respect to mathematics? Discussions in the theme group placed special emphasis on computer systems evolving from artificial intelligence (AI) research and technology that attempts to model students' knowledge of mathematics and behavior in mathematical situations, as well as making use of it in instruction.

In this section we will discuss examples of AI-influenced activities. Particular questions addressed are the following:

- What can we learn form AI computer models about the nature of mathematics learning?
- In what ways can AI technology assist instruction and promote cognitive development?

The central idea of the theoretical study of "artificial intelligence" is to preceed on the basis of the conjecture that every aspect of learning or any other feature of intelligence can in principle be so precisely described that a machine canbe made to simulate it" (original wording of Rockefeller proposal for the Dartmouth conference in 1956). Following the paradigm that the human is an information processing system, AI is the study of how to organize processes to bring about "intelligent" behaviour. Information processing models constitute an approach of extending Piaget's research questions (How is knowledge structured at different stages of cognitive development?) to reach for an understanding of the process of change of cognitive structures which occurs as a result of an individual's active interaction with the outside world. "Learning to generate is learning to understand". The rigid restrictions imposed on computer simulations of cognitive processes require not only the product (i.e. the "intelligent" behavior exhibited), but also the processes giving rise to such behavior to be consistent with observations of human performers.

Since computer simulations of human problem solving processes were carried out by Newell and Simon in the 60's/70's, a growing body of knowledge is accumulating which helps to elucidate processes of cognitive development in light of the information processing paradigm. Techniques have evolved which can serve to make better use of knowledge in specific domains, e.g., expert systems. Such "intelligent" computer programs can, for example, retrieve information necessary to answer questions, perform dialogues with a human partner, assisting individual problems, and analyse errors. They are expected to assist instruction in more than a merely primitive way ("Intelligent computer assisted instruction"). Educational institutions are exploring these possibilities and companies are pushing such technology to the point of application in the classroom. From this evolves the need to discuss the chances, disadvantages, and far-reaching ways in which such technology can affect our children learning mathematics in school or, perhaps in a home computer environment.

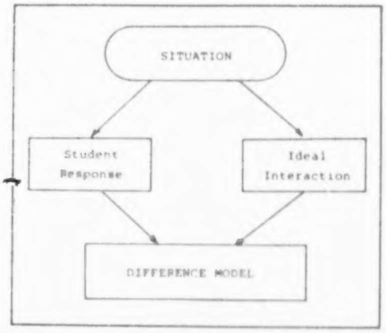
As one example, primarily concerning the first question, Ipke Wachsmuth reported on work that is done at the University of Osnabrück, West Germany. As a central concern of the LAKOS Project ("Logical Analysis of Cognitive Organizational Structures"), a knowldege representation model has been specifed which is implemented in a Prolog machine. A brief demonstration of the model was given in the Technology Summary Session.) The model stresses the organizatoin of knowledge represented in memory and can perform dialogues which are based on individual children's knowledge of specific mathematical domains (e.g., fractions). Based on analyses of interview protocols obtained from long-term clinical research, the hypothesized knowledge structures of individual subjects are modeled in networks which link "knowledge units" that are expressed as logical propositions and stored in indexed memory nodes. In perfroming dialogs with the computer, the behaviour of the model in answering particular queries about mathematical facts can be explored and compared with behaviors that were actually observed with the subject. Situations where certain knowldege is not accessed in a particular context can be modelled was well as confilcts due to misconceptions and inconsistencies. Diffferences occuring between the model's and the subject's actual behavior can lead to a refined understanding of the subject's cognitive structures, since in order to generate certain behavior in the the model, the knowledge units and memeory links need to be specified precisely.

Based on such an hypothesis of what are the current cognitive structures of a pupil, the use of the model is in the position to obtain a diagnosis of the origins of misbehavior, and to evaluate instructional procedures which can bring about progress in the cognitive development of the pupil. With future versions of the model being planned to be capable of self-modifications under interactions in dialogues (i.e., learning), the potential of such a model might even include exploring the effects of alternative teaching procedures in the model before actually intervening in the classroom (though probably at considerable cost). At present, the major pay-off of the model is found in the way it sensitizes the user to the subtleties in the child's understanding of, knowing (or retrieving), and being able to express particular mathematical issues; further, it helps to develop a fine-tuned feeling for performing (teaching or interview) dialogs with the pupil.

The next example concerns the second question of using AI technology in instruction. Betty P Travis (University of Texas at San Antonio, USA) is involved in a research effort on applying Artificial Intellingence techniques to educational software. Here learning to program a computer is viewed as a problem solving task, in which a computer tutor shall interact with the individual student in a manner so as to diagnose the errors in programs written by the student and to lead the student in a Socratic manner to an understanding of the specific errors. For doing this, the system uses the collected knowledge base of experinced teachers of computer programming. As background, the project uses the general model for an ICAI system which is composed of three components:

1 An expert-system component which is charged with the task of generating problems and evaluating the correctness of the student's solutions;
2 A student-model component which is to represent the student's current understanding of the material to be taught and the update of his/her particular error history; and
3 A teaching-strategies (tutoring) component which communciates with the student, selecting problems for him/her to solve and monitoring and critizing his/her performance.

Embedded in a larger famework the core of this approach is to



compare, in a given problem situation, the student's actual response with an ideal interaction generated by the expertise component (see Figure 6). The difference will then be evaluated in order to make a decision about appropriate tutoring strageies. Work is still in a preliminary stage with the main efforts so far being devoted to constructing the expertise component.

In relating the approaches discussed above to the current teaching reality, Ronald H Wenger (University of Delaware, USA) noted that discussion about the content and objectives of the US precollege mathematics curriculum has increased due, in part, to the existence of computer technology. Software such as muMath, PRESS, and MACSYMA can effectively accomplish tasks like equation solving. Such powerful software utilities motivate some to hope that instructional time devoted to such algorithms and procedures can be decreased. While such expectations are often based on exaggerations and little information of the power of the existing software, the scenario does help to focus attention upon serious weakneses in the way mathematics is often taught, and upon weaknesses of many existing textbooks. Virtually all textbooks focus on the "object" or procedural levels of mathematical content and leave discovery of relevant "meta-level" heuristics and strategies to the student. Little improvement is expected unitl new methodologies are created, tested, and used to analyze the higher level forms of cognitive development which mathematicians wish the curriculum to serve.

Wenger also outlined one methodology, the "Global Task Analysis" (GTA), and one project, LP ("Learning PRESS", a PRolog Equation Solving System"), which are complementary efforts aimed at a diagnosis of relevant instructional objectives and which use equation solving as the prototypic domain (at the precalculus level in the United States or the A-level in Britain). The GTA leads to a grammar describing the heuristics, strategies, and procedures hypothesized to be used by the expert when confronted with a mixed list of equations. LP is a Prolog computer program developed in the Aritificial Intellingence Department at the University of Edinburgh which extends the earlier equation solving program, PRESS, to include means of inferring new solution methods from worked examples. In particular, these efforts illustrate the importance of heuristic and other "meta-level" strategies and skills, and they can help to clarify the relevant component skills and their relationships to diagnosing difficulties at strategic and at procedural levels. Some references annotated by Wenger which are relevant to the specififc focus of this work include (Silver, 1984), (Bundy, 1983), and (Carry, Lewis, & Benard, 1980). In addition, a brief and critical summary with respect to the artificial intgelligence theme is found in (Snow, 1984).

It is expected that this work will have implications for the evaluation and design of textbooks and will provide useful foundations for building student models (possibly instantiated in computer simulations) which help to clarify the relationships among concepts and skills used by successful solvers. Furthermore, one hopes to obtain insights as to how much emphasis should be given to the algorithmic skills currently so heavily emphasized in the curriculum, in light of the fact that now computers can often carry out such tasks more skillfully than many students. Further, these efforts will help to identify those algebraic skills which are important for using powerful computer utilities with understanding. Finally, we note that it is hoped that such models can help to obtain a more precise notion of "cognitive structure" which would provide a basis for a more profound understanding of the precess of building-up such structures, i.e. cognitive development.

2.3.4. Possible disadvantages connected with the use of technology

Loss of basic skills and deprivation from natural enviornments were among the disadvantages noted in discussions in the theme group. The fact that the computer might be overused, misused by inexperienced teachers, used for inappropriate topics, or to repeat things which can be done in other ways were viewed as undesirable. Tendencies to "simulate" real-life teachers and an over-reliance on mechanical teaching systems were thought of likewise. The premature pushing of some software projects to classrooms reminded some participants of occurences in the new math movement.

Although some of these remarks were expressed with sometimes great emotional involvement of participants, little evidence was given in terms of findings substantiating these views, though this does not preclude such substantiation in the future. Results of research directed to this issue should be collected in order to obtain a clearer notion of the existence, or non-existence, of disadvantages in the use of technology.

Concerning the question of a possible loss of basic skills through the use of technology, a reference was found of a recently published 3-year study with several thousand subjects, grade levels 7 to 9, which does not substantiate disadvantages with respect to computing skills (Wynands, 1984, p.25): There is no substantial difference in computing skill between the groups of subjects who used calculators from 7th to 9th grade and the control groups who never used calculators. Another reference (Sheingold, in press) includes some remarks related to the aspect of deprivation and inappropriate computer uses which we quote here: "...the greatest source of concern about having microcomputers in classrooms for young children, is that the microcomputer activites will supplant the many activities children do with 'real' materials. ... There is not doubt that working with materials is important for young children, and it would be unimaginable, not to speak of absurd, to have a microcomputer replace the water table, block corner or pet rabbit. What seems to underlie this concern is a sense that the microcomputer innovation has a life of its own proceeding at an intense, unstoppable pace. ...this view implies that the technology will take over, that what teachers do or believe will mot matter. Whatever research knowledge we have on this issue suggests quite the opposite - that what school systems and teachers do with computers - what they use them for, how they interpret them, how they present them to children - has an enormous effect on what happens in a particular system or classroom. ... The technology does not have a life of its own, nor does it stand on its own.

Very probably, it will depend to a great extent on the teacher whether the use of technology is disadvantageous to the cognitive development of children. But it also appears that great differences

BEST COPY AVAILABLE

exist in the quality and suitability of educational software. It would seem that approaches not emphasizing interaction or individual explorations, as well as CAI programs that rely more on rote that on reason, are counterproductive to rather than enhancing of cognitive development. One thing seems sure: Monotonous work which does not provide for self-directed engagement will not foster creativity nor will it contribute to cognitive development of mathematics learners.

2.3.5 Summary

It was realized that technology will play an increasingly important role in the learning and teaching of mathematics. There was accord in the feeling that students may develop a different perspective on mathematics and on learning mathematics as a consequence of working with the computer. However, it is not yet clear what cognitive structures are being developed via experiences stemming from technological stimuli. The computer may provoke approaches which are new and promising but we do not yet know enough about how to capitalize on it. There is doubt as to what technology or how mnuch will be needed. Examples have demonstrated that technology can be very useful, depending on how it is used. While it should not necessarily replace any other effective mediator of cognitive development, the computer may certainly play an important complementary role. Being in the early stages of exploring modes for the use of technology, there is a need for us to search for a "balanced" use of computers in teaching and learning mathematics.

The opinion expressed most frequently in the theme group discussions was that computers can help children in visualizing mathematics and help them learning to think and becoming reflective about their own thinking and acting. Perhaps we could say that the theme group was unanimous in this feeling.

An opinion also expressed repeatedly is that the benefits from using the computer in mathematics learning can be doubly advantageous in that mutual progress is achieved in both operating the computer and mastering subject matter. For example, is was said that interweaving studies in computer topics with mathematics topics can be beneficial, or that a programming language can be learned effectively in the process of solving mathematical problems with the computer, or that when students conceptualize their own programs (e.g. for geometry demonstrations), the acquisition of programming skills will be connected with educational objectives in the subject area and vice versa. This seems to us an important point, in addition to the beneficial influence of the computer on visualizations and on learning to think as mentioned earlier.

Concerning the way in which technology can contribute to a clearer picture of the processes involved in the cognitive development of mathematics learners, it seems an important point that insights can be obtained about the development and differentiation of cognitive strategies via the learners' progressive mastery of symbolic systems provided by a computer. Ultimately desirable is an understanding of the development of cognitive structures in learners' minds in order to understand the origins of their behavior and strategies and be able to account for these in instruction. The use of artificial intelligence techniques in modelling cognitive structures and their development, if complemented/guided by empirical work, may be a promising approach that deserves further research attention.

While the group feels that new technology can be used in many ways in mathematics teaching and thereby may take on a new perspective on mathematics and mathematics learning, it appears necessary to take note of posssible disadvantages in order to maximize the value of technology. Teachers need to know how to handle the microcomputer - its value may to a great extent depend on the teacher and the subsequent interaction of the teacher and the student. At the same time, the group feels that we need to learn more about how to embed what appears beneficial in use of computers into classroom-usable form.

If the development of certain cognitive structures and strategies is what happens in the intellectual development of humans that has long term implications in mathematics learning, then what activities seem to most enhance cognitive development and what cognitive strategies do students need to combine with the use of computers? We did not come up with the answer(s), but we feel we have pulled together ideas and research which move us a little in the direction of (i) understanding better the use of technology in cognitive developpment of mathematics learners, and (ii) kowing more about uses of technology for better understanding of the processes involved in children's cognitive development.

2.3.6 References of Theme Group Contributions

J P Becher: The microcomputer as a tool in problem solving

P Campbell: The young child in a microcomputer environment

E Cohors-Fresenborg: Representations of mathematics and cognitive styles

R B Davis: Frames, metaphors, and computer-assisted instruction

W L Fischer: Computer and transformational geomtry

Ruth Hutter: Using the computer in classroom teaching of high-school and undegraduate mathematics - Concept development, not drill

D K Sinha: Computer education in a developing country: Some conceptual and cognitive aspects

B Travis: A computer tutor for error diagnosis in student programming

P Saengcharoenrat: Computer Based Learning in Thailand

J Wachsmuth: the role of artificial intelligence research and technology for the learning of mathematics

R Wenger: Diagnostic uses of computers - some metaphors and methodologies from cognitive science

G Wheatley: Computers and calculators as tools for exploration and problem solving

K Yokochi & H Okamori: Developing preschool and elementary school children's mathematical idea through educational software

Other References

T F Banchoff (1978): Computer Animation and the Geometry of Surfaces in 3- and 4-space. In Proceedings of the International Congres of Mathematicians Helsinki, Finland, pp.1005 -1013.

J P Becker & K Pederson (1983, June): "Integrating Problem Solving into mathematics teaching." (Proposal funded by the Illinois State Board of Education (ISBE): Springfield, IL). Southern Illinois University at Carbondale Illinois.

A Bundy (1983): The Computer Modelling of Mathematical Reasoning. London: Academic Press

L Burton (1979): The Skills and Procedures of Mathematical Problem Solving. Report of an S.S.R.C.-Sponsored Project at the Polytechnic of the South Bank, London S.E.1, England.

P F Campbell, G G Fein, E K Scholnick, R E Frank and S S Schwartz (1985, April): Initial Mastery of the Syntax and Semantics of Logo. Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, Illinois.

L Carmony, R McGlinn, J P Becker and A Millman (1984): Problem Solving in Apple Pascal. Rockville MD: Computer Science Press

L R Carry, C H Lewis and J E Bernard (1980): Psychology of Equation Solving: An Information Processing Study. Department of Curriculum and Instruction, University of Texas at Austin.

E Cohors-Fresenborg, M Griep, I Schwank (1982): Registermachines and Functions - A Schoolbook Introducing the Concept of Function on the Basis of Algorithms. Osnabrücker Schriften zur Mathematik, U 22E, 22LE, Osnabrück, West Germany.

E Cohors-Fresenborg, C Kaune (1984): Sequential Versus Conceptual - Two Modes in Algorithmic Thinking. In B Southwell, R Eyleand, M Cooper, J Conroy and K Collis (Eds): Proceedings Of The Eighth International Conference for the Psychology of Mathematics Education. (pp. 261-267), Sydney.

E Cohors-Fresenborg (in press): On the Representation of Algorithmic Concepts. In F Lowenthal et al. (Eds): Pragmatics and Education. New York: Plenum Press.

R B Davis (1984): Learning Mathematics: The Cognitive Science Approach to Mathematics Education. Kent, England: Croom Helm; Norwood, New Jersey: Ablex.

G G Fein, P F Campbell and S S Schwartz (1984): Microcomputers in the Preschool: Effects on Cognitive and Social Behavior. Journal of Applied Developmental Psychology, manuscript submitted for publication.

D Gentner and A L Stevens (1983): Mental Models. Hillsdale, New Jersey: Erlbaum.

G A Goldin (1983): Levels of Language in Mathematical Problem Solving. In J C Bergen and N Herscovics (Eds). Proceedings Of The Fifth Annual Meeting Of The North American Chapter Of The International Group For The Pyschology Of Mathematics Education, Montreal, Canada (pp. 112-120).

W Metzler, W Beau and A Uberla (1985): Anschaulichkeit bei der Modellierung und Simulation dynamischer Systems. In H Kautschitsch and W Metzler (Eds): Mathematische Modelle und Visualisierungen. Schriftenreihe Didatik der Mathematik, Universität für Bildungswissenschaftern in Klagenfurt. Wien: Hölder, Pichler u. Temtsky; Stuttgart: Teubner.

National Council of Educational Research and Tranining (1984). CLASS - Computer Literary and Studies in Schools. Report of the National Workshop on Computer Literary Curriculum held at NCERT on 26-27 March 1984, New Deli, India.

S Papert, (1980). Mindstorms: Children, Computers and Powerful Ideas New York: Basic Books.

H O Peitgen & P H Ritcher, (1984). Morphologie Komplexer Grenzen - Bilder aus der Therorie dynamischer Systeme. Bremen, West Germany: Universität Bremen.

P Saengcharoenrat, (1984). CBL in Thailand: The Promise of the Smile. Section Paper presented at ICME 5.

A H Schoenfeld, (1983). Problem Solving in the Mathematics Curriculum. MAA Notes, No.1, Washington, D.C.: Mathematical Association of America.

K Sheingold, (in press). The Microcomputer as a Symbolic Medium. In P F Campbell & G G Fein (Eds.), Young Children and Microcomputers. Reston, Va: Reston Publishing.

B Silver, (1984). Using Meta-level Inference to Constrain Search and to Learn Strategies in Equation Solving. Ph.D. Dissertation, Department of Artificial Intelligence, University of Edinburgh.

R E Snow, (1984). Human and Artificial Intelligence: Some English-Dutch Contrasts. European Scientific News 38-2, 63-66.

I Wachsmuth, (1985, April). Logical Analysis of Cognitive Organizational Structures - The LAKDS project. Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, Illinois.

J L Wright, & A S Samaras, (in press). Play and Mastery. In P F Campbell & G G Fein (Eds.), Young Children and Microcomputers. Reston, VA: Reston Publishing.

A Wynands, (1984). Rechenfertigkeit und Taschenrechner - Ergebnisse einer dreijährigen Untersuchung in den Klassen 7-9. Journal für Mathematik-Didaktik Vol 5, No 1/2, 3-32.

K Yokochi, (1983). Educational Meaning of Personal Computers - Around Some Development of Instructional Software. Bulletin for Mathmematics Education Study (Mathematics Education Society of Japan), Vol.24, No.1-2, 35-51

K Yokochi & S Machida, (1982). Personal Computer and Program-Calculator for Mathematics Education. Tokyo: Gyosei.

TELEVISION, VIDEO & FILM

Television and Video Report

3.1 Film and Video in Mathematics Education
Chair/Reporter: Jack Koumi (UK)

3.2 Film and Video about Mathematics Education
Chair/Reporter: Jack Jaworski (UK)

Group Members:
Darrly Burrows. Australia
Ruth Hubbard. Australia
Robert Johnston. Australia
Meg Luckie. Australia
Linley Pascoe. Australia
Christine Sutherland. Australia
Melvyn Weston. Australia
William Barton. New Zealand
Kozo Kasetani. Japan
Nitsa Movshovitz-Hadar. Israel
Gerald Porter. USA
Connie Knox. UK
Amy Wootten. UK
Jack Jaworski. UK
Gail Hood. Australia
Richard Jasiowski Australia
Murray Lamshed. Australia
Bruce Nulty. Australia
Terry Smale. Australia
Shirley Tyler. Australia
Kenneth Voges. Australia
John Anderson. New Zealand
Alan Parris. New Zealand
Hiroko Kakiya. Japan
Chisato Yamaguchi. Japan
Ada Sherer. Israel
Micele Emmer. Italy
John Rigby. Singapore
Des Wilson. UK
Jack Koumi. UK
Chair/Reporter: David Roseveare. UK

3 Television, Video and Film

3.1/3.2 Film and Video in Mathematics Education
Film and Video about Mathematics Education

3.1.1 Summary

The majority of participants in these two groups opted for Group 3.1. Group 3.2. started with just two members, but grew in size daily. Members of both groups attended general viewing sessions and film/video-related Project Presentations - Emmer (Italy), Hadar (Israel), Koumi and Jaworski (BBC Open University Production Centre, UK). The two groups combined for the final session.

A fundamental problem facing those who wish to discuss film and video (f/v) material is that it needs to be viewed, in real time, in advance of the discussion itself. When a f/v lasts for a total of only 5 minutes, the viewing can reasonably be incorporated into the time allocated for discussion but many educational f/vs are of 20-25 minutes duration: to show only short excerpts will give some impression of style and content but it cannot adequately give a feeling of the overall structure, nor of whether it appears to have achieved its stated aim. This is particularly true when the f/v forms an integral part of a larger teaching scheme, as with distance education.

In attempting to overcome this problem, we encouraged participants to view f/v material at ICME 5 outside the Theme Group sessions but this was not easily achieved in practice and we have to acknowledge that our expectations exceeded our achievement. Participants have since asked for more predigestion of material by the group organisers and it may be that we should have narrowed considerably the range of material under discussion.

On the positive side, this was the first ICME to have included specialist working groups devoted exclusively to film and video. (1984 also saw the First International Congress on Mathematical Movies, held at the University of Turin.) An encouragingly large number of participants were either already involved in f/v production or were shortly about to start work in this field.

Jack Koumi (UK) drew up a suggested checklist of some 35 points to be borne in mind when devising an educational TV programme (not necessarily mathematical), grouped under 9 main headings: Hooking, Signposting, Scripting, Coherence, Sympathy with viewer, Active (mindful) learning, Attuning (making the viewer more receptive), Range of viewer ability/preference, Ending. Examples (good and bad) were taken from BBC Open University programmes to illustrate these points.

One participant called for more discussion of material known (by its producer) to be poor. One such example was bravely brought to Group 3.2 and subjected to scrutiny: the (amateur) producer has since made a tape/slide presentation of the same material (for teacher education), claimed to be more effective but lacking the portability of videotape.

Because utilisation procedures are different from one country to another, we tended to discuss production details within each f/v rather than the place of f/v within a particular educational structure. Not all countries have national or even regional curricula, not all production countries are exclusively concerned with educational material, some teachers rely heavily on broadcast material, others have access to good video distribution systems, and so on.

We are exploring the possibility of setting up an international Newsletter and maybe and exchange scheme whereby producers of mathematical f/v (and their advisors) can view each other's material between one ICME and the next. This would be distributed to the 30 participants of our two ICME 5 Working Groups, plus other interested parties, identified by our participants. One topic which might be discussed between ICMEs is the clear distinction which emerged between tertiary level material and that for schools (primary and secondary). The examples of the latter which were shown, from Israel, BBC (UK) and Granada (UK), concentrated much more on motivating students than did the tertiary level material from the Open University of the United Kingdom; that is, the OU material was much denser cognitively. Participants felt that one reason for this is that the material for schools was intended as a classroom resource, to be introduced and followed up by the teacher; in contrast, OU programmes are intended for distance learning by individual adult students without a teacher. This rationale for the distinction in styles may or may not be valid; perhaps school f/v should increase its cognitive element while tertiary f/v should increase its motivationial element?

A guest at our final session was Robert Davis (USA) who presented a 20 year old film recording of a real-time videotaped lesson conducted by himself. It is a tribute to the production procedure that this class-obesevation material is as valid today as in the 1960s; the fact that we all became involved in the teaching procedure suggests that the video recording had done its job well, despite our efforts to examine it in its own right. The medium is definitely not the message, but it can be an extremely effective messenger.

CLASSROOM DYNAMICS

4.1 Feedback to the Learner

Group Members:

Grant Barnett. New Zealand
Deborah Clayton. Australia
Brian Doig. Australia
Roderick McLean. Australia
Edward Sawyer. Australia
Jeffrey Wakefield. Australia
Edor Oscarsson. Sweden

Gary Bitter. USA
Stanley Daniels. Australia
James Finlay. Australia
Dennis O'Brien. Australia
John Scheding. Australia
Jon Coupland UK
William Ebeid. Egypt
Karsuhiko Sato. Japan

Co-ordinator/Reporter: Jon Coupland UK

4.1.1. Discussion Points

4.1.2. Group Report
4.1.2.1 Introduction
4.1.2.2 Definition
4.1.2.3 Classification
4.1.2.4 Feedback Systems
4.1.2.5 Conclusions

4.2 Roles of the Microcomputer in the Maths Classroom

Group Members:

Gunter Alle. West Germany
Lynette Bloom.Australia
James Cross.Australia
Jefferey Gazzard.Australia
David Haimes.Australia
Leslie Holten. Australia
Lesley Hurburgh. Australia
Michael Quinn. Australia
Carolyn Ehr. Australia
Stephen Moresh. USA
Lars Bergstrom. Sweden
Rosemary Fraser. UK
Marie Louise Hocquenheim France
Jacqueline Peyrot. France
Piet Human. South Africa
Ronald Van Hamelsveld New Zealand

Peter Barcham.Australia
Edgar Briedis.Australia
Graham Ferres.Australia
Brian Goodwin.Australia
John Hoggan. Australia
Peter Hough. Australia
Brenda Nettle. Australia
Stephen Rice. Australia
Johnny Lott. USA
Connie Widmer. USA
Douglas Butler. UK
Rachel Hebenstreit. Australia
Serge Hocquenheim. Australia
Roger Maurin. France
Elizabeth Oldham. Ireland
Shoichiro Machida. Japan
Richard Wah. Fiji

Co-ordinator/Reporter: Rosemary Fraser UK, Carolyn Ehr USA.

4.2.1 Discussion Points

4.2.2 Group Report
4.2.2.1 Introduction
4.2.2.2 EUREKA
4.2.2.3 GREENGLOBS
4.2.2.4 PIRATES
4.2.2.5 EQUATIONS DE DROITES
4.2.2.6 SNOOK

4.2.3 Contributed papers

4.3 Learning Enviroments

Group Members:

Heather Chapman. Australia
John Hunter. UK
Bruce Nulty. Australia
Sharon Dugdale. USA
Richard Phillips. UK
Peter Samuels. Australia

Chair/Reporter: Richard Phillips

4.3.1 Discussion Points
4.3.2 Group Report
4.3.3 Contributed Papers

4 Classroom Dynamics

This group as a whole was interested in exploring the different ways in which the microcomputer with a range of software might influence the learning and teaching in school mathematics classrooms. In essence we started with the view that the classroom is a complex human situation and that the microcomputer introduces a new dimension to the existing interactions. It is also a perturbation to the current situation which might provide us with further insights of avenues of communication that could enhance our current practices.

In order to focus discussion and provide the opportunity to simulate classroom interactions specific programs were studied. Three groups were convened each with a different brief related to classroom dynamics. To share the outcomes, experiences and ideas that emerged?, it will be necessary to 'set the scene' for the reader. Extracts from the original discussion paper will describe the starting point of explorations and activities for each group's workshop. A group report summarises the workshop activities leading into comments of general interest. A selection of papers related to each group are also published.

4.1 Feedback to the Learner

Group Members:

Grant Barnett. New Zealand
Deborah Clayton. Australia
Brian Doig. Australia
Roderick McLean. Australia
Edward Sawyer. Australia
Jeffrey Wakefield. Australia
Edor Oscarsson. Sweden

Gary Bitter. USA
Stanley Daniels. Australia
James Finlay. Australia
Dennis O'Brien. Australia
John Scheding. Australia
Jon Coupland UK
William Ebeid. Egypt
Karsuhiko Sato. Japan

Co-ordinator/Reporter: Jon Coupland UK

4.1.1. Discussion Points

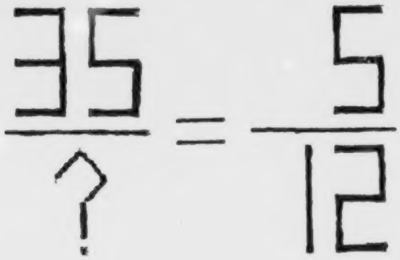
Is it possible to classify different types of feedback to the learner? Here are a few possibilities.

(a) Comparing solutions with others and discussing.
(b) Getting examples marked with verbal or oral comments.
(c) Testing solutions in some practical manner - exploring models and systems.
(d) Checking progress regularly before proceeding to the next step.
(e) Working towards a given solution ie access to the 'answers'.

Once a list of different ways in which the learner receives feedback on his progress is agreed we could consider the frequency with which this feedback is given and also the demand on the learner that the feedback might impose. Thus we might explore manner, frequency and demand of the various types of feedback.

Using two examples of feedback where computer programs have played a part a few questions can be raised.

Consider a screen which displays two equivalent fractions with one of the 4 figures missing eg.



$$\frac{35}{?} = \frac{5}{12}$$

The ? appears randomly in any of the 4 possible positions. After a short time delay the screen shows the missing number.

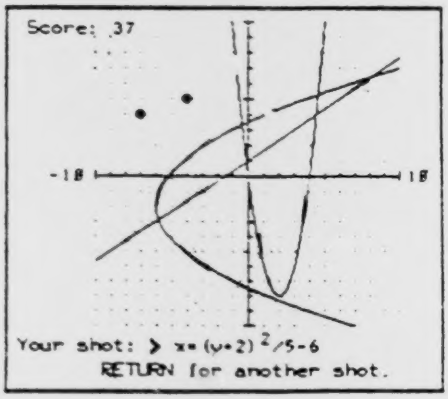
These images are from a computer program AUTOFRAC (1) which produces a relentless succession of problem followed by solution. Pupils are challenged to mentally produce the missing number. Two levels of difficulty may be set before the program sets off. The first level allows you to control the range of the numbers that will appear and the second level allows you to set the time delay that you have to solve the problem (i.e., time before the computer displays the answer).

So the **manner** of feedback is to supply the answer for you to check out your own solution. The **frequency** of the feedback is immediately after each problem (timed very precisely). The **demand** of the feedback is to find the quickest way to solve the problem. It has been observed that the effect of this program is that almost 100% of viewers cannot resist trying to get the answer. Because of the time demand they use quite different methods to those most commonly taught and indeed different methods from one example to the next. Finally if the time is reduced such that they have difficulty in 'beating the computer', most people stop trying altogether. Another possibility with this program is to cause it to generate 'incorrect solutions' from time to time. This adds additional demand to the student in that he also has to recognise when these incorrect solutions appear. How would this add to the activity?

It would seem that the manner and demand of the feedback certainly influences the cognitive processes and the student's choice of solution. Could we employ this rather unusual type of feedback to advantage in other situations?

Many CAL programs have their roots - consciously or unconsciously - in Skinnerian learning theory. This tends to treat feedback and reinforcement (or reward) as inseparable. The smiling face or the jolly little message - "Well done, Merald!" - are both your reward and the information that you are correct. AUTOFRAC is interesting in that is gives you feedback but totally eliminates external rewards. The computer cannot praise you because it never knows if you are right or wrong.

In the GREENGLOBS (2) program the computer displays a co-ordinate grid on which it graphs any equations that are typed in by the student. Thirteen green objects, each about 0.7 units in diameter, are scattered about the grid, hence the name of the activity, 'Greenglobs'. The goal of the game is to explode all of the green globs by hitting them with graphs, specified by typing in equations. If a student's shot misses the expected target, the graphic feedback (display of the student's graph) gives diagnostic information needed for the student to debug his or her ideas about graphs. Perhaps the graph was too wide or too steep or upside down. The student can test his or her diagnosis immediately on the next shot. For each shot the first glob hit is worth one point. The second is worth two points, the third is worth 4 points and so on. For example a five-glob shot would score 1+2+4+8+16 for a total of 31 points.



```
Your shot: > x=(y+2)^2/5-6
        RETURN for another shot.
```

Here the manner of the feedback is a graphical picture of how well you are doing. The frequency of the feedback is under the student's control, the demand of the feedback is to diagnose the errors and adjust accordingly. Also scoring included in the feedback moves the student towards solutions that he may well not have attempted had the scoring algorithm been other than it is.

A system to explore, that responds with such information and encouragement, must have a great deal to offer us.

References

(1) AUTOFRAC
Jon Coupland, College of St Mark & St John, Plymouth, England.

(2) GREENGLOBS
Sharon Dugdale and David Kibbey, University of Illinois, USA.

4.1.2 Group Report

4.1.2.1 Introduction

The group spent some time viewing software, both to explore the varying types of feedback provided and to provide a common software base to support discussion. Throughout the sessions, attention was always strongly focussed on the use of technology to enhance mathematical education rather than to the mere exploration of computers. The results of our deliberations can best be summarised by a definition, some dimensions for classification, and a diagram of the dynamics of a feedback system.

4.1.2.2 Definition:

Feedback is a response, from the computer or other learners, to the learner's interaction with them.

4.2.2.3 Classification:

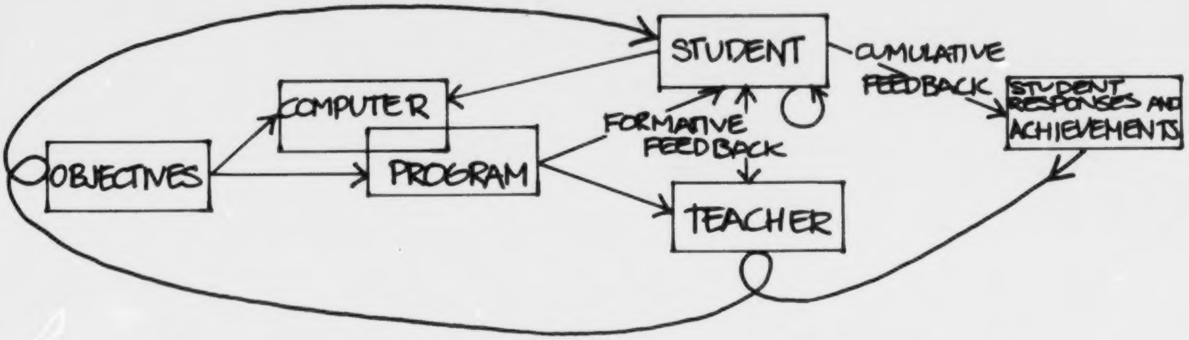
The following dimensions for classification and the examples given are designed to highlight aspects of the interactions between the computer and the student which will need to be considered when we focus on possible styles of feedback to the learner. There is no attempt to make value judgements; the aim is to give an indication of the spectrum of possible feedback. Although we use the term 'classification', the field of computers-in-mathematics-education is in its infancy and fast evolving. Developments in hardware and software, and our understanding of their implications for teaching and learning, will continue to grow and change.

(a) Psychological effects of feedback. Behavioral - cognitive
Attitudinal - feedback being competitive and providing satisfaction.
(b) Mode of presentation of feedback/Sensory modality.
Sight - text, graphics, colour, special effects, animation
Sound - music, speech, special effects
Touch - special peripherals
(c) Instructional mode - feedback varies according to:
Drill-and-practice, tutorial, enquiry, simulation, text, game, computer-managed instruction, problem solving, wordprocessing, laboratory, guidance, demonstration.
(d) Interactions ('A/B' here indicates 'A to B') between different parties. computer/learner, computer/teacher, teacher/learner, learner/teacher, teacher/computer/learner, learner/learner.
(e) Degree of help of feedback.
Limited/extensive, tutorial, diagnostic/remedial.
(f) Predictability of the feedback.
Unexpected/predictable, anticipatory/non-anticipatory.
(g) Frequency of the feedback
Spaced (rarely receiving), massed (frequently receiving).
(h) Machine capability or limitations with regard to feedback.
(i) Correctness of responses.
Correct, non-committed, incorrect.
(j) Type of user considered or not considered by feedback offered.
Special needs of the learner, etc.

It was felt that the desirable features of feedback would include graphics, sound, colour, animation, humor, variety, clarity, engaging, positivity, simple responses and the promotion of other activities. Undesirable features would include lengthy texts, mathematical symbols, complex keyboard response and the need to move to other screen displays.

It should be noted that, as with all curriculum material, the educational aims have top priority. For practical reasons, some desirable features will frequently be excluded and some undesirable features included. However, we do feel that due attention should be paid to this rather idealistic categorisation.

4.1.2.4 Feedback Systems - One Approach



This diagram (Fig. 1) highlights the difference between formative (or immediate) feedback occuring during the actual use of the computer and the cumulative feedback of the total student involvement with the teaching package. Additionally, the outer loops emphasise the use of feedback in the development process - indeed, the continual process of development - undergone by teaching material.

4.1.2.5. Conclusion:

As indicated above, the group considered mathematical education rather than computing. Indeed, the general feeling was that, as with more traditional materials, the majority of the feedback would take place between the student and teacher, and student with fellow students. However, our analysis should provide useful guidelines for software designers, publishers and users.

4.2 Roles of the Microcomputer in the Mathematics Classroom

Group Members:

Gunter Alle. West Germany
Lynette Bloom.Australia
James Cross.Australia
Jeffer y Gazzard.Australia
David Haimes.Australia
Leslie Holten. Australia
Lesley Hurburgh. Australia
Michael Quinn. Australia
Carolyn Ehr. Australia
Stephen Moresh. USA
Lars Bergstrom. Sweden
Rosemary Fraser. UK
Marie Louise Hocquenheim France
Jacqueline Peyrot. France
Piet Human. South Africa
Ronald Van Hamelsveld New Zealand

Peter Barcham.Australia
Edgar Briedis.Australia
Graham Ferres.Australia
Brian Goodwin.Australia
John Hoggan. Australia
Peter Hough. Australia
Brenda Nettle. Australia
Stephen Rice. Australia
Johnny Lott. USA
Connie Widmer. USA
Douglas Butler. UK
Rachel Hebenstreit. Australia
Serge Hocquenheim. Australia
Roger Maurin. France
Elizabeth Oldham. Ireland
Shoichiro Machida. Japan
Richard Wah. Fiji

Co-ordinator/Reporter: Rosemary Fraser UK, Carolyn Ehr USA.

4.2.1. Discussion Points

Can we identify the different responsibilities and rules in the classroom that the teacher, students and computer program can assume? A crude taxonomy of the possible rules might be:-

Manager - managing the learning
Explainer - explaining mathematical concepts, ideas, solutions
Task setter - setting up the task(s)
Fellow pupil - working on the task(s) together
Counsellor - helping others working on the task
Resource - available for the provision of further information

The teacher, student or computer program may play these rules. It is of interest to consider which of these roles we, as teachers, and the students, as learners, most commonly adopt and how effective this is. Linking this to Paragraph 243 of the Cockcroft Report (1) it would seem that most mathematics teachers tend to use the Manager, Explainer and Task setter roles much more than the Fellow pupil, Counsellor and Resource roles (2). The latter three roles are necessary to sustain problem solving and investigatory activities. Using the computer program as a teaching assistant very often allows the teacher to move into managing and explaining roles. The students also are very often able to imitate both the role of the teacher and/or the computer program. In this way they can be led naturally into activities that demand higher level skills.

Effective 'teaching assistant' type programs are seen by the students to have a personality of their own - the teacher is able to gain great benefits from this perception, very often being able to become a 'fellow pupil' or 'counsellor'. The following lesson illustrates many of the points made above. It employs the following taxonomy.

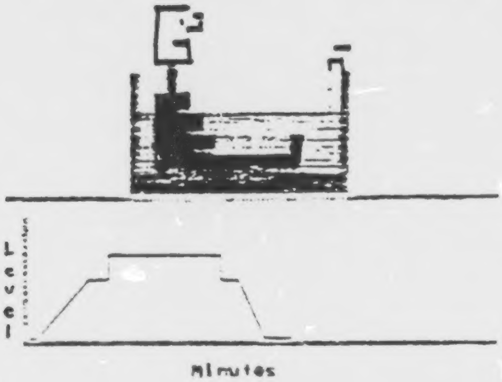
T M E C F R are the 6 roles. Task setter, Manager, Explainer, Counsellor, Fellow pupils, Resource and t, c, p, indicate whether teacher, computer, or pupil are playing these roles.

e.g. Tt = teacher is the Task setter
Mt = teacher is Manager
Ec = computer is Explainer
Mp = pupil is Manager

Lesson 5 **Jim with EUREKA**

EUREKA (3) is a program which develops skill in the interpretation and sketching of graphs. The screen displays a cartoon picture which shows a man taking a bath; the various elements of this process are controlled by the user using simple key presses. Under the picture is displayed a graph of the water level in the bath against time - it is semi-quantitative with labelled axes but no scales. Either the cartoon or the graph may be suppressed in the display. And sequence may be replayed and there are a number of built-in examples which may be called by the user.

1



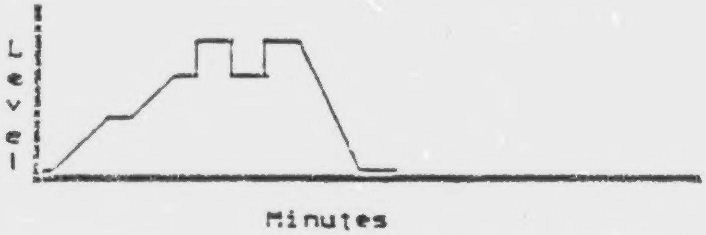
The lesson began, as lessons with EUREKA usually do, with the teacher using the program to demonstrate and explain what is involved in a cartesian graph. Both cartoon and graph are "on". The teacher manages a sequence, with comments from the class - plug in, taps on, taps off, man in, sings (some teachers comment here), an attempt to get the man out which failed because the singing is continuing, stop singing, man out, plug out. Pupils watch with interest.

Traditonal didactive activities (d) and roles, Tt, Mt, Et enlivened by the vivid scene setting Ec of the program, whose humorous and very silightly risque personality maintains a high interest level. Pupils are passive.

Turning off the cartoon, leaving only the graph, the teacher uses the stored example 2 to prodcuce a graph on the screen. Still in exposition to the whole class, there is a lively discussion of what is happening. The example is replayed with the cartoon on, confirming the essential correctness of the class's conclusions.

Here the acitivity demands on the pupils changes, to problem solving (p) with high level skills (h), though still at an easy level. The micro is thus task setting Tc; the teacher still manages Mt the task but discusses the various possible interpretations of the class with definite signs of the counselling role Ct.

2



Next the teacher provides a worksheet divided into four sections, and asks each child individually to write in the top section a story. (They are invited to use the screen prompts "man in" etc, if they want - they all do). They are then to draw in the second section the graph that represents what occurrred. This produces lively interest and some discussion, with mixed but generally favourable results.

BEST COPY AVAILABLE

Each child is then told to fold over the paper so the story is hidden, and to exchange with their neighbour who must then interpret the graph, writing the story as they see it in section 3 of the worksheet.

Changing back the author then comments in section 4 of the worksheet on the interpretation in section 3, and on the accuracy of his graph in section 2. This works fairly well - though earlier conversations suggest other channels of communication than the graph may have played their part. The activity focuses detailed attention on the graph as a means of communication and discussion is lively.

The computer is now switched off, having played its role as vivid game setter Ec, which persists by implication throughout. Teacher is task setter Tt but management is now shared with the pupils Mp, whose learning activities are higher level (h) involving image building, analysing, abstractions, and symbolising (s).

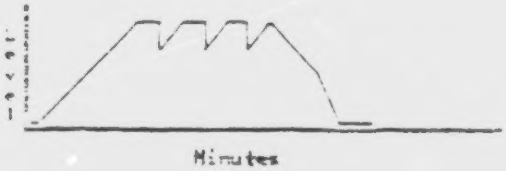
A similar balance of roles, with high level activities (h) now also bringing in a problem solving (p) element.

Here ralfecting and (h) activity which is particularly elusive, is the centrepiece, with elements of proving involved through justifying what has been drawn on the graph.

A continuation of this high level activity is promised, involving additional resources contributed by the teacher, and an intriguing but difficult challenge is provided in example 6. Its interpretation will doubtless lead to later amusing discussions.

The teacher finishes by promising other types of graphs for the next lesson, and by providing a worksheet of example 6 for interpretation for homework.

6



Many programs are being developed that can become excellent 'teaching assistants'. Pictureware (4) from France uses the graphics facilities to great advantage, the Greenglobs (5) program mentioned above is just one of a very rich collection of material from the University of Illinois. One of the major problems of reviewing this type of teaching material is the huge variety of ways in which different teachers will use it. This is why it is well worthwhile considering the role of teacher, student and computer program and not to concentrate solely on what the computer screen is doing.

References

(1) COCKCROFT REPORT 'Mathematics Counts' London:HMSO 1982

(2) Learning Activities and Classroom Roles, Shell Centre for Mathematical Education, University of Nottingham, England.

(3) EUREKA
Richard Phillips, Hugh Burkhardt and Malcolm Swan, Shell Centre for Mathematical Education, University of Nottingham, England.

(4) Pictureware Imagiciels, Institut Nationel de Reserche Pedagogique, Paris, France

(5) GREENGLOBS
Sharon Dugdale and David Kibbey, University of Illinois, USA

4.2.2 Group Report

4.2.2.1. Introduction

A very practical workshop was undertaken by this group. We divided into six subgroups, each group explored a range of programs, selected one program and prepared 'lesson plans'. These 'lessons' were then presented to the whole group and discussions of the 'classroom diagnosis' were promoted.

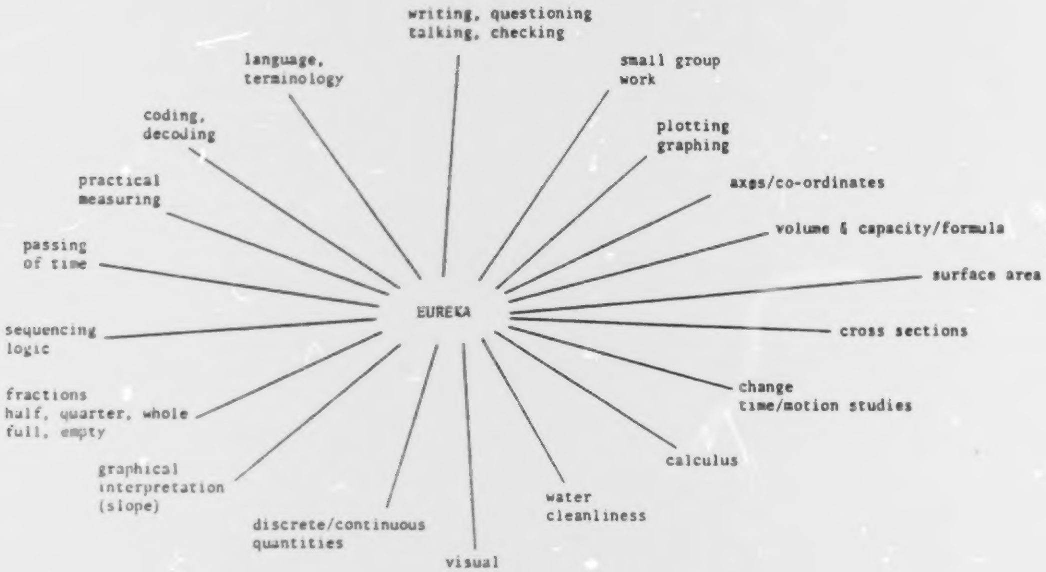
We report the ideas offered with each of the programs explored in this manner and relate back to the roles classification that was described above.

We recommend that where possible readers obtain the software used and explore its potential with their own classes then consider how their own experiences compared with the ideas others had after this brief opportunity to review materials at ICME5. One of the major problems with "dynamic" materials of this type is that they need to be reviewed "in action", before any attempt to classify their uses is made. Each teacher will adapt their use to particular aims and objectives. Our premise is that "children interacting with each other and their teacher with interesting problematic situations and with the microcomputer may construct a powerful form of mathematical thinking."

4.2.2.2. EUREKA Program

The group exploring the possible 'lessons' that could 'use' the EUREKA program (as described above) offered a very wide range of possibilities for pupil activities. The detailed suggestion covered the age range from 5 years old to 15 years old.

The diagram below summarises the possibilities:


EUREKA
writing, questioning
talking, checking
small group
work
plotting
graphing
axes/co-ordinates
volume & capacity/formula
surface area
cross sections
change
time/motion studies
calculus
water
cleanliness
visual
discrete/continuous
quantities
graphical
interpretation
(slope)
fractions
half, quarter, whole
full, empty
sequencing
logic
passing
of time
practical
measuring
coding,
decoding
language,
terminology

The program was felt to be particularly supportive in stimulating group discussion and for younger children was used to promote creative writing. The oral discussions and activities would help teachers to connect the mathematical concepts to the real world and also encourage symbolism skills. Other uses mentioned were:

(a) extracting variables
(b) making changes from a qualitative view to a quantitative view.
(c) considering relationship between two variables

Conclusions

(a) Classroom Roles
EUREKA is particularly good as **Explainer/Scene Setter** and in helping to **Manage** and **Taskset**. Pupil's learning activities can be promoted away from the computer as well as with the computer. Teacher experiences varies modes of use according to age range and ability of children. "Omission design" ie being able to not show animation or graph etc is a powerful catalyst to "active" learning rather than "passive" interactions.

(b) In this area of graphical interpretation programs of this type are powerful in helping students build up translation skills; words $\rightleftharpoons$ graphs symbols $\rightleftharpoons$ graphs situation $\rightleftharpoons$ graphs. It is possible with such support to introduce pupils to such concepts at an earlier age.
(c) It was felt that EUREKA was a useful model to software designers.

EUREKA Richard Phillips, Hugh Burkhardt and Malcolm Swan, Shell Centre for Mathematical Education, University of Nottingham, England.

4.2.2.3. GREENGLOBS program

This program is described in 4.1.1. It basically requires the user to define functions that will 'hit' as many of the 'Greenglobs' (randomly generated) as possible . The scoring system increases exponentially and the 'best' situations we strived for then use to explore.

The group considering this program made some general comments:-

In the Cockcroft Report "Mathematics Counts" there is a recommendation concerning the desirable range of learning activities:"

Paragraph 243 Mathematics teaching at all levels should include opportunities for:

(a) exposition by the teacher
(b) discussion between teacher and pupils and between pupils themselves
(c) appropriate practical work
(d) consolidation and practice of fundamental skills and routines
(e) problem solving, including the application of mathematics to everyday situations
(f) investigation work

With appropriate software the microcomputer can act as a very valuable aid for the above learning activities. We consider now the program GREENGLOBS could be used effectively in a series of lessons. We feel that the program is most useful as an aid for activities (b), (c), (d) and (f).

Lesson Objectives for 17-18 Year Olds Who Have Acquired Basic Graphic Skills for Linear and Quadratic Equations

(Note: The program GREENGLOBS could be used with younger students subject to restrictions on the type of equations to be entered. Also the program could be used quite effectively with tertiary students for a wide range of equations.)

Objectives

By the end of these lessons students should:

(a) be able to determine the equations of a line, circle or parabola containing a selection of points.
(b) have recognised the standard forms for the equations.
(c) have developed an understanding of the relationship between an equation and its graph eg $y = a(x - h)^2 + k$ - turning pt (h,k).
(d) explored the effect of transforming standard shapes.
(e) have the opportunity of exploring a challenging situation.
(f) have the opportunity of sharing their experience with fellow pupils in a small group setting.
(g) revised equivalent forms of equations.

Conclusions

(a) Classroom Roles
The teacher is able to move into fellow pupil role - hitting the 'globs' is a fun activity for all ages. The program is task setter often teacher has set the scene (explained the rules). Program responds with graphical feedback - children can correct and observe. Program will also record best solutions and children can learn from other's attempts.

(b) Once again this program can provide a large range of activities for a broad age and ability span. It is easy to control in the classroom and promote some novel strategies not normally encountered in the more traditional approach to this area of functional graphing.

(c) Special design features worth noting are the form of serving and the recording of the best situation.

GREENGLOBS Sharon Dugdale and David Kibbey, University of Illinois USA.

4.2.2.4 PIRATES program

PIRATES is a treasure hunt carried out in either a two or three dimensional grid. The treasure is found by a process of successive guesses. Following each attempt, the program responds with some helpful information. The nature of these clues can be selected by the teacher from: compass directions, simple getting warmer/colder messages, bearings (and elevations in three dimensions), vectors, and distance to or from the 'treasure'. Many different strategies emerge in the various 'games' and a great variety of mathematical activities is prompted in the classroom.

Two groups considered the PIRATES program and devised several possible 'lesson' plans. The following comments were made:

(a) It is a program that could help teachers promote a lively problem solving sessions.

(b) Technical skills required 'to play the game' are simple but the strategic skills required to search effectively for the treasure are high level skills.

(c) Setting the level of strategie demand needs to be carefully considered by the teacher. If the demand level is too low the motivation drops, if the demand level is too high students 'guess' rather than think strategically.

(d) Program is suitable for large groups, small groups and individuals.

(e) The program does not record any 'co-ordinate points'. The pupil's take responsibility for recording the data, organising it and solving the problem.

Conclusions

(a) Classroom Roles
PIRATES assumes the task setter role and helps considerably with the management role. The teacher is able to become a fellow pupil in a very natural way and also a resource for help and a counsellor for discussion.

The pupils are very actively involved but care must be taken to set a suitable demand level or the hunt may be reduced to a guessing game. There are over 120 different treasure hunts that can promote quite a varied set of strategies to be explored.

PIRATES ITMA Collaboration, College of St Mark and St John, Plymouth, England.

4.2.2.5 EQUATIONS DE DROITES program

The program allows the exploration of the parameters of straight lines. Very clear graphics are used and the two forms of equation $y = ax + b$ and $ax + by + c = 0$ are available.

The group that considered this program were impressed with the accessibility of the graphics. The program is extremely simple to operate so children can concentrate on the behaviour of the graph and are not distracted by computer controls eg for the form $y = ax + b$ pressing the 'A' button on the keyboard increments A by one each time, ie produces rotation, pressing the 'B' button increments B, ie produces translation. The children can explore this and discuss what is actually happening.

One of the lessons considered is discussed below. The form of equation $y = ax + b$ is set. The children just press 'b' several times. Each time the program responds by increments the value of 'b'. Thus a family of straight lines appears. It is also possible to decrement the value of 'b'. The children are asked to discuss what is happening. In a similar way they can explore 'a'.

The activities encourage the children to explore the system themselves and to come to some conclusions. After a suitable interval students are asked to write a brief summary stating what they saw happening and asking them to make some generalisation about the mathematical symbols contained on the screen and relating it to the representation of the graph on the screen. The program can also be presented in class mode for discussion. This will probably be most useful after the children have had a chance to work on it on their own. Finally the teacher will recapitulate the main points that came out of the investigation.

Conclusions

(a) Classroom Roles
The program provides a mathematical object to explore. It acts as a resource; the children are asked to observe and comment. The teacher's role is counsellor and finally explainer. The dynamic nature of the program responding to the pupil's commands motivates interest and illustrates relationships in a set of vivid images.

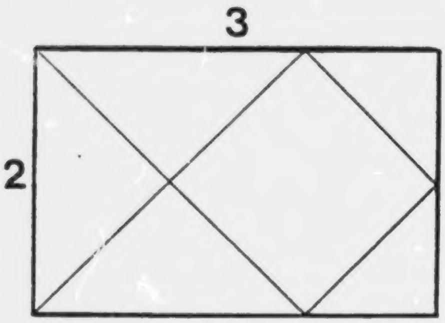
EQUATIONS DE DROITES: Serge and Marie Louise Hocquenheim IREM, Paris, France.

4.2.2.6 SNOOK Program

SNOOK supports the investigation of a ball moving round a rectangular table until it falls into a corner pocket. The ball can be chosen to set off at any desired angle or gradient. A wide range of discussions can be promoted at many different levels. The program provides a powerful illustration of inductive reasoning leading to mathematical proof.

Pupil activities: investigation, analysing, generalising, discussing, proving.

The following work sheet suggests some investigation:



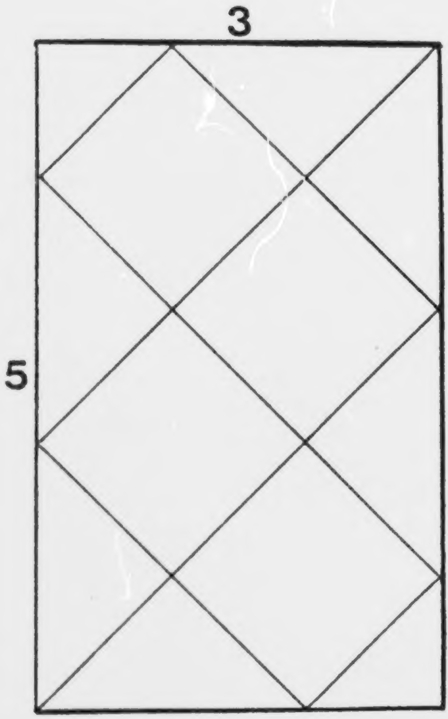
What rule connects the length and width of the snooker table with the number of times the ball touches the sides?

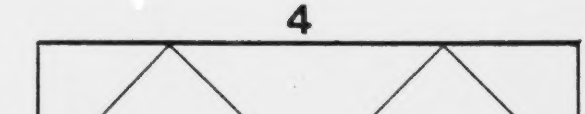
According to your rule how many times will the ball touch the sides on a 2 by 2 table?

Does your, possibly revised, theory provide an appropriate solution for a 200 by 300 table?

What rule connects the length and width of the snooker table with the corner in which the ball eventually stops (assume the ball always leaves from the bottom left corner)?

What rule connects the length and width of the table with the number of regions it is divided into?





The group exploring this program tried various approaches for classroom use. General discussion then followed and finally a discussion on the proof of the problem was stimulated. Consideration of the proof brought the discussion to an interesting level for the participants. It was presented with consideration given to the way in which children of this age group would in fact be able to follow and was not dependant on formal algebra to any great extent. A critique of the program was then presented and included the following points:

(a) Suitability and applicability. Not directly applicable to any standard curriculum but suitable as a problem solving activity, also an an enrichment or extension activity. Suitable for ages 11+.
(b) The vertical/horizontal lines are distracting - should be available as an option - can have some rectangle with different measurements.
(c) Would have preferred the order width, then height rather than height then width.
(d) The program clearly demonstrates "shortest" distance and "symmetry".
(e) When the angle (gradient) is changed the program is unforgiving and students may become very frustrated (if gradient is irrational).
(f) The on-screen directions are not sufficiently explicit. Also 0 - 90 is not understood as (0,90) but (0,90)??
(g) Given that this problem is worth solving, it's much quicker to do it with the computer than with pencil and paper.

Conclusions

(a) Classroom Roles
SNOOK is the system to explore, it provides data as requested by pupils. Thus it adopts the resource role. The teacher can become fellow pupil or counsellor quite easily. Children have to try simple cases, build up tables, look for patterns and generalise. Teacher may decide to leave the problem solving activity very open or may set a variety of structured investigations. For older children the further extension to consider mathematical proof is possible. The program offers discussion in algebra, number and geometry and can provide input at various points in the curriculum.

SNOOK: Problems with Patterns and Numbers (full version), Joint Matriculation Board, Manchester, ENGLAND.

Micros in the Mathematics Classroom (short version), ITMA Collaboration Longman, 1982.

4.2.3. Contributed Papers

Computer Assisted Teaching;
Rachel Hebenstreit FRANCE

Applications of the Microcomputer to Mathematics Teaching;
Lynette Bloom, Geoffrey Combar, James Cross AUSTRALIA

Teacher, Student and Computer Roles in Mathematical Classroom;
Shoichiro Machida JAPAN

Roles of the Microcomputer in the Mathematics Classroom; Experiences at Settle School;
John Kitchell, Jack Upsall ENGLAND

Computer Assisted Teaching

Mrs Rachel Hebenstreit
"Pictureware" Group
IREM - Université PARIS 7
2, place Jussieu
75005 PARIS FRANCE

"What is an idea? It is a picture painted in my brain. Are, therefore all your thoughts pictures? Truly: because the most obstract ideas are just the sequences of all the objects I have seen. I have ideas only because I have pictures in my head".

Voltaire (Dictionnaire philosophique)

ABSTRACT:
The vast majority of the packages existing in C.A.L. is meant to be used by students in a more or less autonomous way i.e. without the presence of the teacher.

Very few do exist, which are meant to be used by a teacher, during his course to improve his teaching. If a computer can assist a student in his learning activities, why can't the computer be used to assist the teacher in his teaching activities?

The research described is a tentative to answer positively the above question in the teaching of mathematics to children in the age 10 to 15.

At all times, pictures have been used as an illustration of the discourse in the pedagogy of mathematics.

It is by the analysis of a sequence of pictures, that certain properties emerge; it is by comparing drawings, examples and counter-examples that hypothesis appear in the mind of students, followed, in many cases, by the will to verify these hypothesis.

TEXT:
For a long time, the mathematics' teacher has been using chalk of different colors to illustrate his course. Usually the teacher draws on the blackboard outlines, curves, straight lines in a hasty and approximate fashion, because his course lasts only fifty minutes.

What is significant for the teacher is not always so, for the student the "implicit" properties of a drawing remain too often "non-explicit" for the student.

Children between 10 and 15 who do not generally have highly developed abstraction abilities, appreciate clean, clear and colored graphic representations, which display successive varied or particular cases of the same notion.

The teacher, therefore, prepares a set of transparencies as an animated set and uses the overheadprojector to illustrate his course.

But the students, even if they ask questions and make comments on the course, are unable "to act" on the shown pictures or to modify them.

Therefore the idea to use a microcomputer, with color graphics abilities, during mathematics courses for projecting animated pictures, modifiable upon request with well chosen colors to illustrate particular properties of a given theme, has been chosen as a research subject inside the Institute for Research on Education in Mathematics (I.R.E.M. - University Paris 7 - FRANCE).

The produced "pictureware", as an introduction, or a resource, or an illustration, or a conclusion of a course, is a specific pedagogical material of the "Teaching Assisted by Computer".

In the academic year 1981-1982 the co-operation between mathematicians and computer scientists from University and teachers from secondary education inside I.R.E.M. has resulted in the production of a number of software-packages as well in Algebra as in Geometry.

About a dozen "picturewares" have been completed on the GOUPIL 2 (a French microcomputer) already used inside I.R.E.M. for other research subjects.

The specifications of the machine have been used at their best, including the contraints (definition 256 x 256) and the advantages (seven bright colors).

Two screens are used simultaneously: a text-screen and a colour-graphic-screen.

The themes studied are the mappings in a plane and the study of the linear and affine functions.

Other projects are under way.

The design and the pedagogical aims may change for each package, depending on the subject. Here are two examples:

The observation of animated pictures, first simple and then complex, or first complex and becoming simpler will bring students to a collective discussion with a halt on a picture, going back to a past picture and with all possible modifications in dimensions, forms, distances, upon request form the students.

The co-ordinates of two points can be chosen by the students who can request, step by step, the construction of the straight line going through the two points and ask for the equation of the straight line (which they will have been requested to calculate beforehand)

This group discussion initiated and managed by the teacher, aims to lead the students towards the discovery of the properties and specific relations of the visualized mathematical situations.

The richness of this form of collective course seems intuitively very great, but the best appropriate possible themes and their way of presentation remain to be studied.

Other graphic media may be used for these collective courses "illustrated by computer" depending on their availability as a giant television screen or a number of television monitors in the class, to improve the visibility by every student.

One may hope that further progress in technology will bring on the market more powerful computers (definition, number of colors etc ...) at prices compatible with what educational institutions can afford.

All packages have in common the following pedagogical aims:

- to have the greatest possible number of students participate actively and specifically the most passive, or the most timid, by asking them to propose their own data or changes.

- All the programmes are written in such a way, that all mathematics teachers who wish so, are able to use them, even if they have no knowledge at all in computer science. For instance:

- any picture can be stopped by a simple stroke on any key of the keyboard.
- when a question is asked on the test-screen, the way the answer has to be typed is always given on the screen.
  no typing error can block or destroy the running programme. In case of any typing-error, the computer answers with a question. Therefore, anybody, teacher or student can type on the keyboard.

"Simplicity of use" has been a constant preoccupation for the authors of "pictureware".

The pedagogical intentions and the choice of the theme have led, first of all, to scenarios which were discussed by the group and often modified to use at their best the possibilities of the computer and/or to by-pass its technological constraints.

The detailed script of each programme has also been discussed, step by step, with successive changes, to keep at their best the legibility of the pictures, the juxtaposition of colors, the right places of the pauses, the "windows", the calculations, the pedagogical value of such and such a possibility or a simplicity of use.

The design of each package has necessitated two or three meetings of the group, each week, with very lively and sometimes passionate discussions.

The net outcome is that the design of a package is a very time-consuming task.

Most of the packages are now under experimentation in different classes, but have been used for too short a time to lead to any significant result.

Computer Assisted Teaching is a new concept. Very little has been done in that area because the main stream is oriented towards teaching teachers programming languages or an "author language" to develop packages, for use by the students, in the absense of the teacher.

Much remains to be done, to make available to teachers, packages which they can use to improve their teaching, without any prerequisite knowledge in computer science. Further experimentation will tell us, if we have or not taken approach to C.A.T.

Applications of the Microcomputer to Mathematics Teaching

Lynette M Bloom, Geoffrey A Comber and James M Cross
(Department of Mathematics (Nedlands), WACAE Perth WA 6009)

1. INTRODUCTION

There is no doubting the potential of the microcomputer as an aid to the mathematics teacher. This is supported by the findings of T J Fletcher as outlined in his discussion paper (7). With appropriate software the microcomputer can be effective in a variety of classroom modes. It can help to illustrate particular mathematical concepts both through the use of professionally written programs and the development of short programs written collectively by the teacher and his/her students.

We have been developing appropriate microcomputer software for use by mathematics teachers and students at the secondary and tertiary level. The software was designed to meet the needs of our own classroom teaching. The programs are both for teacher use, including Computer Aided Demonstration (CAD) and the use of the computer as an 'electronic blackboard', and for use by students in the tutorial mode. We have found that the aspects of the microcomputer most relevant to the mathematics classroom are portability, (color) graphics, and animation.

It is desirable that at least one microcomputer be available for use in the classroom when needed. If access is restricted then the use of the microcomputer is both limited and artificial. The microcomputer should be seen as an integral component of the classroom. We use a large monitor and trolley which can be easily wheeled into the classroom. The potential of even one 'stand alone' microcomputer is considerable.

We have found high resolution color graphics and animation to be essential in achieving the full potential of the microcomputer in mathematics. This too is supported by T J Fletcher (7). High resolution graphics enable the construction of elaborate mathematical diagrams with comparatively little time and effort.

Our software has been written for use on the BBC Acorn B microcomputer. The areas in which we have been involved include transformation geometry, graphing of functions, calculus and statistics.

In what follows we shall indicate briefly how we use several of our programs in particular classroom situations. For a more general description of some of our software and areas of use see (1).

2. TRANSFORMATION GEOMETRY

We have written several programs in this area concerned with the rigid transformations (translation, reflection, rotation) and also shear and dilation. They are written at a variety of levels. In one program a rigid transformation is selected from a list while in another the choice of transformation is via an appropriate matrix input.

One of these programs considers all of the above transformations and allows for composite as well as single transformations. The user inputs four points for the initial shape and then selects from a menu of transformations. Fig. 1 shows for example the choice for the initial input of points (2,1),(1,2),(2,3) and (3,2) and a choice of dilation with respect to the point (1,1) with scale factor of -2.

transformation geometry, graphing of functions, calculus and statistics.

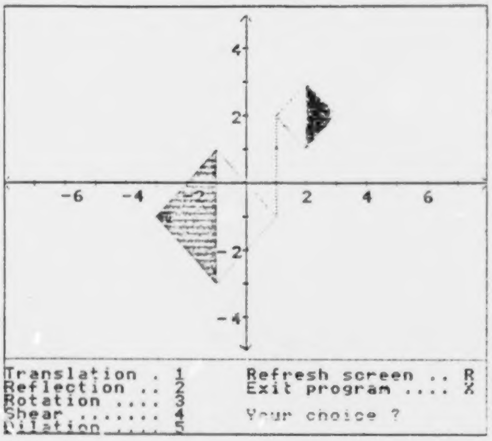


Fig. 1

The different shadings in Fig. 1 correspond to the different colors used. The initial and final images are shown but it is not possible here to convey the animation on the microcomputer. Intermediate steps are provided indicating the motion of the transformation.

The above example would be difficult to present without the use of the combined color, animation and speed of the microcomputer. With the option to perform successive transformations many properties can be developed without resorting to numerous and time consuming diagrams on the chalkboard or overhead projector.

3. GRAPHING EQUATIONS

With appropriate software the microcomputer can be used to graph functions and relations of the teachers or students choosing. It can be used as an electronic chalkboard or an aid in teaching appropriate graphing skills.

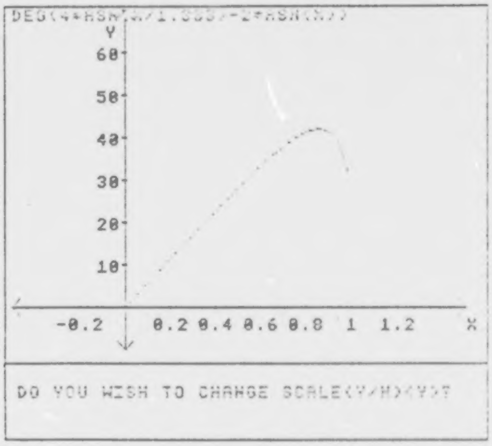
We have six programs dealing with this topic. These programs are:

a) Graphing standard functions (STDFNC)
b) Graphing arbitrary functions (GRAPHS)
c) Graphing polar equations (POLAR)
d) Graphing parametric equations (PARAM)
e) Graphing functions via a transformational approach (TFGPH)
f) Guess the function (GUESS)

The first of these, STDFNC is menu driven providing the graphs of standard functions including polynomial, trigonometric, exponential, logarithmic, greatest integer, absolute value, square root and inverse proportion functions. This program is useful when examining the properties of a standard function.

The programs GRAPHS, POLAR and PARAM are much more general allowing users to input arbitrary functions, polar and parametric equations. In the program GRAPHS the user chooses the domain of the function and the ranges of interest on the x and y axes. Once the function has been graphed the user can change the axes scales, thus enabling a 'zoom' effect for any region of particular interest. Applications of this program include graphing simple and complicted functions, solution of equations, limits of functions and the examination of turning points and points of inflection. (An examination of the use of GRAPHS in determining particular limits of functions is given in (4)).

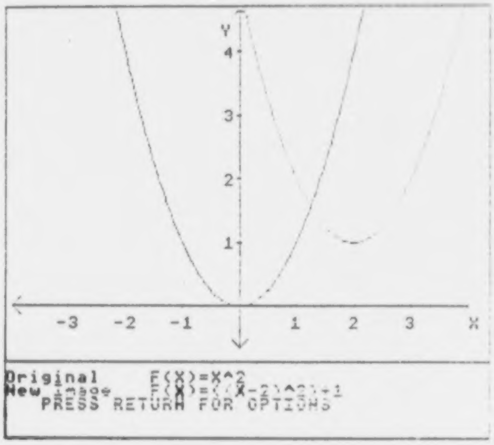
Quite often the examination of 'real' problems leads students to functions which are much more complicated than those they have encountered previously. Access to a microcomputer enables them to quickly examine the graphs of those functions. An example of one such function occurs in a model of a rainbow. The graph of the function obtained is given in Fig. 2.



```
DEG(4*ASN(X/1.333)-2*ASN(X))
DO YOU WISH TO CHANGE SCALE(Y/N)<Y>?
```

Fig. 2.

The program TFGPH is intended as an aid in the teaching of the transformational approach to graphing functions. On selecting from a list of a standard function the user can select from a range of transformations. On selecting a transformation the transformed graph and the corresponding equation are given. The user can transform the original or last image graph. For example the graph obtained by translating the function $f(x) = x^2$ two units to the right and one unit upwards is given in Fig. 3. The program can be used by students to check their graphs and to discover how transformations of graphs affect their equations. A more detailed examination of the use of the microcomputer in teaching the transformational approach is given in (5) and the results for some particular functions can be found in (2).



```
Original     F(X)=X^2
New image    F(X)=((X-2)^2)+1
   PRESS RETURN FOR OPTIONS
```

Fig. 3.

The program GUESS can be used as an exercise set either in conjunction with TFGPH or separately. The user selects a function category and is presented with the graph of a function. The user must then try to determine the corresponding function. The computer draws the graph of the user's choice and if the graph is incorrect it invites the user to continue guessing. This program has the features of a computer game and has proven to be successful both in the instructional and tutorial mode.

We have noticed an improvement in students competence in and attitude to graphing functions as a result of using the above program.

4. CALCULUS

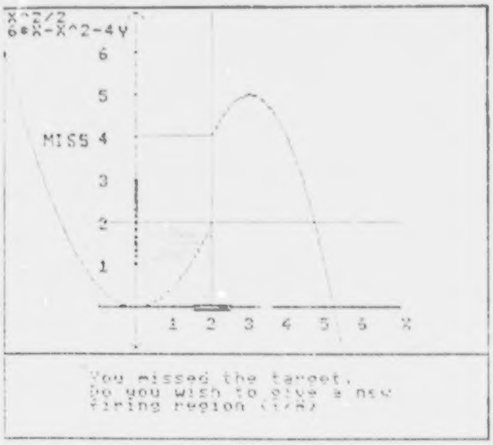
The microcomputer has the potential to be an effective aid in the teaching of both concepts and skills in Calculus. Three of our programs relating to concepts are:

(a) Limits of functions (LIMIT)
(b) Tangent Line to a Curve (TANGENT)
(c) Area under a curve (AREA)

The program LIMIT has been written to motivate the formal definition of a limit. Colour, speed, animation and sound are important features of this program. After being presented with a graph of a function the user must guess the limit of the function at a particular x value. The user must then determine a 'firing' region on the x axis such that f(x) hits a specified 'target' region containing the guessed limit. The user can clearly see if f(x) hits the target. Different firing and target regions can be selected at any time. An example for

as shown in Fig. 4

The user has guessed the limit to be 2. It can be observed that a 'firing' region cannot be obtained for the given 'target' region and hence the limit cannot be 2.



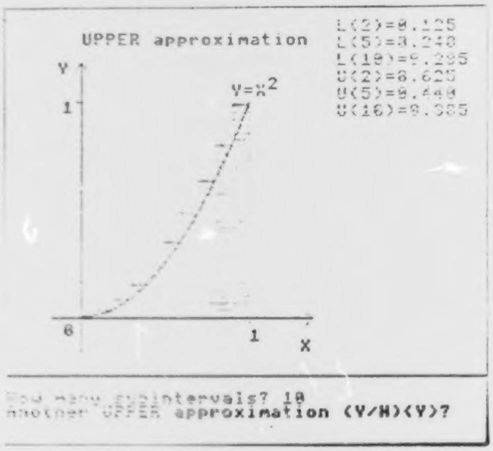
```
You missed the target.
Do you wish to give a new
firing region (Y/N)
```

Fig. 4

The program TANGENT demonstrates the construction of the tangent line as the limit of appropriate secant lines. The user selects a point on a curve and a sequence of smaller and smaller increments. Both the graphs of the secant lines an their slopes are provided.

This program can be used to construct tangent lines at a variety of points on the curve. This enables students to explore the connection between the slope of the tangent line and properties of the original curve.

The program AREA deomostrates the apprximation by the areas of appropriate rectangle, of the area bounded by the curve $y = x^2$, the x axis and the lines $x = 0$ and $x = 1$. The user specifies the number of upper or lower rectangles. An example of an approximation is given in Fig. 5. The speed and accuracy of the microcomputer enables the students or teacher to select a great many approximation during the lesson.



```
L(2)=0.125
L(5)=0.240
L(10)=0.285
U(2)=0.625
U(5)=0.440
U(10)=0.385
```

```
How many subintervals? 10
Another UPPER approximation (Y/N)<Y>?
```

Fig. 5

5. STATISTICS

The speed, graphics and random number generator of the microcomputer can be used very effectively to simulate typical probability experiments. This should not be viewed as a substitute for the students own first-hand experience, but rather as an extension of that experience for large samples.

We have written programs to simulate a variety of experiments involving the rolling of dice, Buffon's Needle experiment and Monte Carlo estimation of area. In the dice-rolling programs the user can select the number of dice to be rolled and the number of rolls and consider the experiment of adding the numbers on the upturned faces of the dice as well as outcomes related to the binomial distribution. By providing the option of displaying the appropriate normal approximations in all cases these programs can be used to illustrate the Central Limit Theorem. For a more detailed description see (3) and (6).

A second important use of the microcomputer in statistics is the presentation and analysis of data with emphasis on graphical displays. Examples of programs we have written in this area are:

(a) Box and Whisker Plots (BW)
(b) Line of Best Fit (LBF)

The program BW calculates a five number summary for data entered either directly or via a previously prepared data file. The five number summary consists of the maximum, minimum, median, and the upper and lower quartiles for the data. The five number summary is displayed together with its pictorial representation as a box and whisker plot. The user has the option of displaying a box and whicker plot for a single set of data or by using side-by-side box and whisker plots to compare two sets of data. An example of a display is given in Fig. 6.

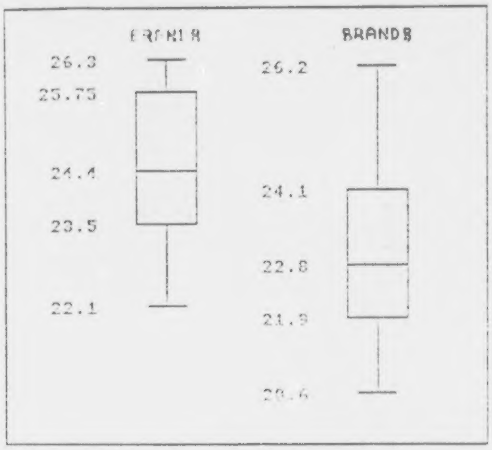
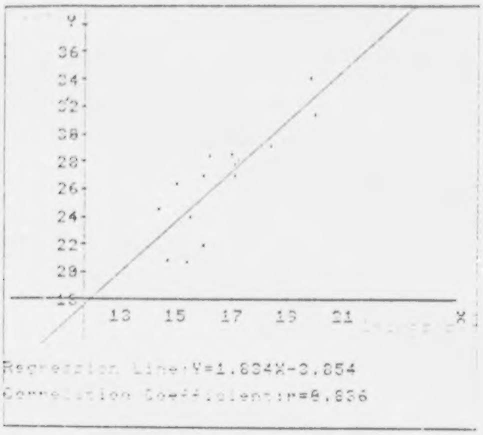


Fig. 6

The program LBF determines the least squares line of best fit and correlation coefficient for given data. The user inputs the data either directly or via a data file, and the corresponding scatter diagram is displayed on the screen. The user can then obtain the line of best fit and its equation and also the correlation coefficient for the data. In addition with appropriately chosen data the teacher can demonstrate the concept of correlation coefficient. An example of the final display comparing the frequency of cricket chirps and temperature in degrees Celsius is given in Fig. 7.



```
Regression Line:Y=1.834X-3.854
Correlation Coefficient:r=0.836
```

Fig. 7

Details regarding this data may be found on page 409 in (8).

CONCLUSION

We believe that the microcomputer has the potential to be an extremely powerful aid to the classroom teacher for a wide range of topics in mathematics. Its versatility in terms of speed, (colour) graphics, animation and portability enables the teacher and students to explore throughout the lesson a variety of mathematical situations, many of which would not otherwise have been possible.

Our experience with the use of the microcomputer in the classroom has shown that with appropriate software and teaching style the microcomputer can result in an improved learning environment with a higher level of motivation. Our assessment of students' ability in understanding and applying mathematical concepts and skills has supported this finding.

REFERENCES

(1) L M Bloom, G A Comber and J M Cross, 1984, The Use of the Microcomputer in the Mathematics Classroom, Computing and Education - 1984 and Beyond, Computer Education of Victoria, ed. by A D Salvas, Melbourne, 234-244.
(2) L M Bloom, G A Comber and J M Cross, 1984, Graphing Through Transformations via the Microcomputer, Sigma, 121, 2-6.
(3) L M Bloom, G A Comber and J M Cross, 1984, Teaching the Binomial Distribution Using Both Student and Microcomputer Simulations, Sigma, (to appear).
(4) L M Bloom, G A Comber and J M Cross, 1984, Two Important Limits via the Microcomputer, (submitted for publication).
(5) L M Bloom, G A Comber and J M Cross, 1984, Use of the Microcomputer to Teach the Transformational Approach to Graphing Functions, (submitted for publication).
(6) L M Bloom, G A Comber and J M Cross, 1984, Using the Microcomputer to Simulate the Binomial Distribution and to Illustrate the Central Limit Theorem, (submitted for publication).
(7) T J Fletcher, 1983, Microcomputers and Mathematics in Schools, Department of Education and Science, United Kingdom.
(8) Richard J Larsen and Morris L Marx, 1981, An Introduction to Mathematical Statistics and its Applications, Prentice Hall.

Teacher, Students and Computer roles in mathematical Classroom
- On an Experimental Trial of CAL in Japan

Shoichiro Machida

DEPARTMENT OF MATHEMATICS
FACULTY OF EDUCATION
SAITAMA UNIVERSITY

URAWA, JAPAN.

We propose a special kind of the learning system which is supported by plural microcomputers assisted a teacher in a classroom. We call this system CAL or Computer Assisted Learning. Through this system we do not aim at the perfect automatic learning system like CAI.

We aim mainly at some kind of group learning dealing with individual difference of students.

When we analyze the interaction of a teacher and students in an ordinary classroom, we are able to pick up the following six critical points in the process of teaching and learning.

(1) presentation of the today's tasks and motivation.
- as task setter.

(2) managing problem solving activities through discussion among students and between a teacher and students, and through works with manipulation.
- manager.

(3) suggestion and explanation to students activities by a teacher.
-explainer, counsellor.

(4) managing cultivation of skills through exercises.
- manager.

(5) summerizing the tasks by a teacher and students themselves.
- counsellor.

(6) evaluating the attainment of the aims and design the next lesson's plan.
- Diagnostician.

This classroom interaction between a teacher and his/her students is very complicated, interdependent and generative.

Recently, increase in the number of students, diverse student's performance and requests and motivation, enlargement of the social needs to the flexible mathematical problem solving ability is forcing us to consider the following problem.

How can a teacher teach many students with diverse performance and requests at the same time and same place to develop the flexible ability of mathematical problem solving?

Our CAL is willing to solve this problem successfully with the harmonious team work of a teacher and micro computers. We will illustrate the role of teacher, students and computers in our CAL.

There are the following five phases in the mathematical lesson. The teacher will change his/her strategy according to each of these phases.

In our CAL system, we will have to consider the system configuration and the role of teacher, students and computers.

Phase 1. Lesson for learning the basic concept and understanding knowledge.
Phase 2. Lesson for acquisition of skills of computation.
Phase 3. Lesson for introducing the synthesized power of the task.
Phase 4. Lesson for testing, evaluating and summarizing student's achievement.
Phase 5. Lesson for illustrating the mathematical power of problem solving in real world.

Example 1. As Computer Resource of subject matter for Phase 3
- Pythagorean Theorem -

The Pythagorean Theorem is one of most important and fundamental theorems for junior secondary school students in Japan.

Pythagorean Theorem

In a right triangle, the square of the hypotenuse equals the sum of the square of the other two sides.

Finding out the proof of this theorem is demanded for the sythesized power of the task and therefore, this lesson is for phase 3.

Teacher divides his/her classroom students into about five small groups and he/she gives them the following task.

Find out the relation between the hypotenuse and the other two sides in a right triangle.
- teacher setting task.

Pupils discuss their ideas. Pupils draw some right-angled triangles and measure the length of three edges of each triangle to estimate the relation.
- student manager

Teacher asks another question to his students.
Are your estimates correct for any right-angled triangle?
Let's consider the proof of the Pythagorean Theorem.
- teacher task setter.

And then, teacher suggests his/her students switch on their computers. Some kinds of Micro-CAI programs of the Pythagorean Theorem are needed. Teacher selects one of the suitable programs according to the student's performance and requests.
- teacher management

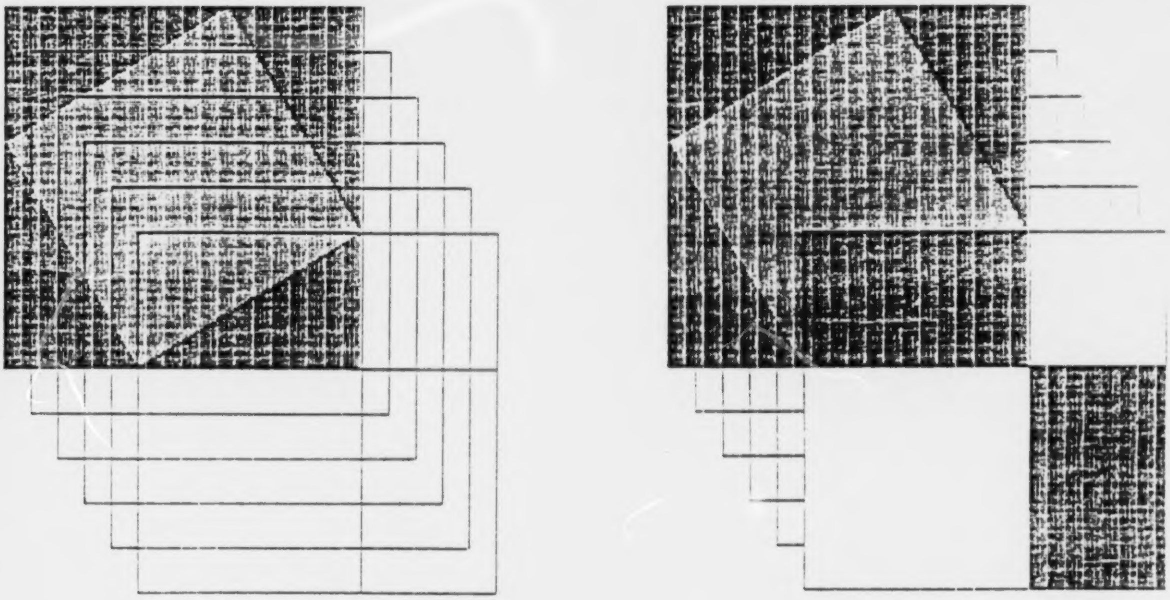
In each group, students learn an example of proof through colourful and operational computer display.

- computer explainer
- teacher counsellor
- student manager.

After learning through computer support, students discuss the way of their proof to complete their own report. At last, the representative of each group publishes the result of their own report to all classmates and received the question and compares with other's way of proof.

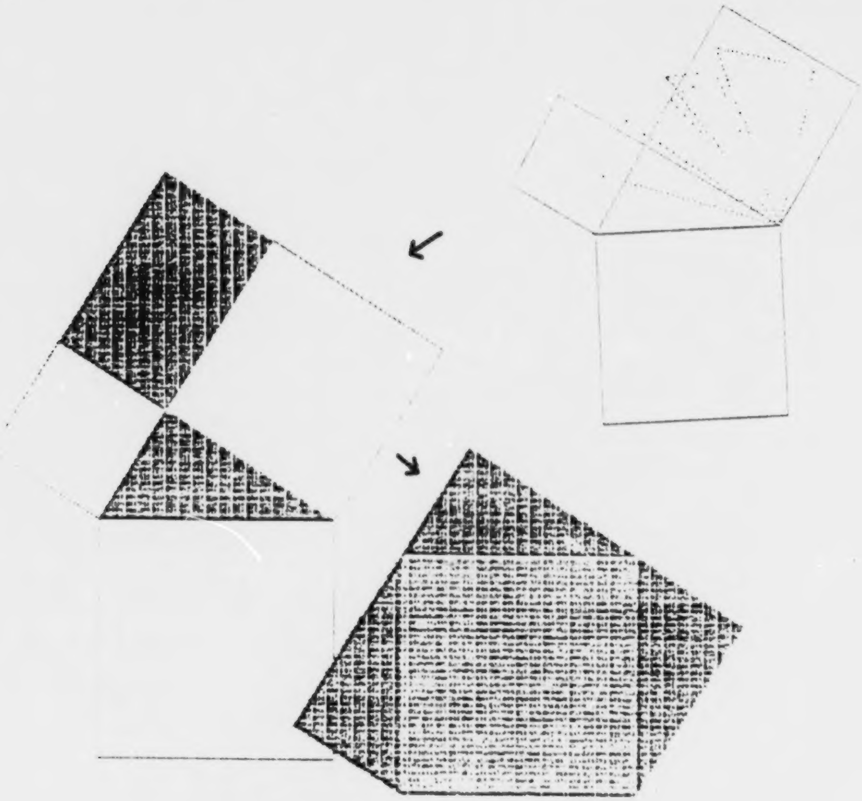
- teacher manager
- computer fellow pupil
- student counsellor, explainer.

PROOF 1.



BEST COPY AVAILABLE

PROOF 2.



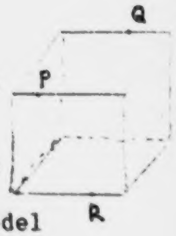
After the lesson, when there were some interesting ideas from students, teachers put their ideas into his/her computer files.
- computer resource.

Example 2. As flexible Manipulation Aids for Phase 3.
- Cut a Cube or a Cylinder -

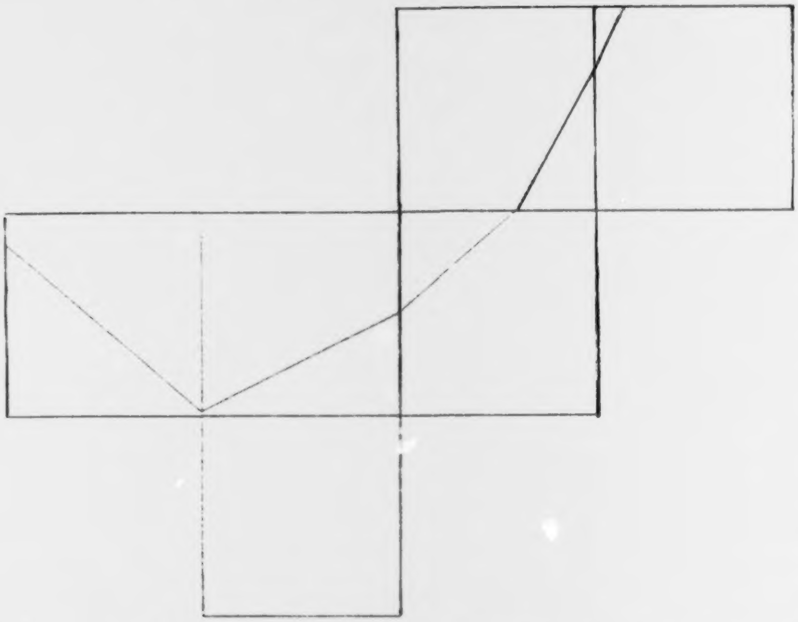
One of the most weak points in the educational use of micro's is that computer display is not able to replace by the role of human hands. To make compensation for this point, we make full use of low cost x-y plotter or plotter printer.

We will show one of the examples.

Task: Complete a sketch of the following cube that is cut by a plane through point P, Q and R.



Computer automatically draws an arbitrary development of the cube included a cut surface on a thick paper according to student's needs. Students cut the development out and make up the paper model of the structure of this task.

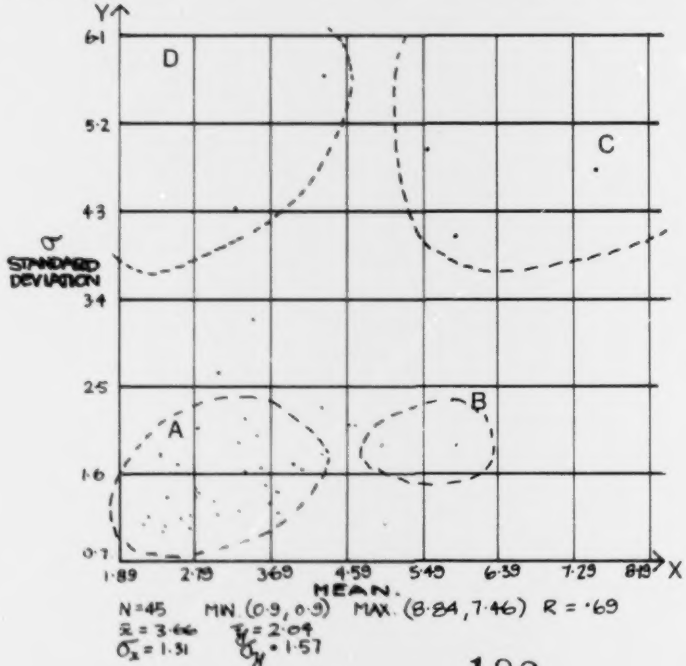


Example 3. As Diagnosis of Student's Performance for Phase 4.
- Stationary Power of Multiplication Facts -

It is very important for CAL to diagnosis the student's performance, because teacher decides his strategy for the next lesson according to the result of this diagnosis.

When we give the drill and practice through computer programs to our students, it is especially important for us to know the staionary power of student's achievement.

Diagnosis of Stationary Power of Multiplication Facts - Each point shows a cartesian co-ordinate (mean of response time, standard deviation of response time) for correct answer of each student. See below.



Very good for group A.
Little more hold on for group B.
More harder work for group C.
Don't be so hasty for group D.

References

1. Shoichiro Machida. (ed.), Personal Computers in Mathematical Education - Application and Program -, Mizu-umi Shobo, Tokyo, 1983.
2. Christopher Smith. (ed.), Microcomputers in Education, John Willy & Sons, London, 1982.
3. Rosemary Fraser, Hugh Burkhardt, Jon Coupland, Richard Phillips, David Pimm and Jim Ridway, Learning Activities and Classroom Roles with and without the Microcomputer. ITMA Collaboration, 1983.

ROLES OF THE MICRO-COMPUTER IN THE MATHEMATICS CLASSROOM

The experience of Settle High School

John Kitchell and Jack Upsall, Settle High School, Yorkshire, ENGLAND.

The mathematics department at Settle High School has a long history of teaching mixed-ability groups. In response to the demands of mixed-ability work staff have been compelled to adopt a flexible and thoughtful approach to their teaching. Significant effort has been put into the search for appropriate ideas, the design of materials, and the development of teaching styles and methods of assessment. Computer Studies has been taught within the department for many years, and for the last 7 years a short appreciation course has been included in the 3rd year mathematics course, taught by all the mathematics staff. This combination of a growing familiarity with computing, has led us to investigate the use of the computer within mathematics lessons.

From the outset it was clear that commercially available programs were largely inappropriate for our needs. Over the years, a particular style had emerged in our production of worksheet materials and we thought it important that the computer programs should reflect this natural style and should be compatible with established standards of language and presentation. We therefore began to write our own, relatively simple, yet robust programs in order to complement and support a variety of existing, successful topics. We recognised that these programs might lack some sophistication but we anticipated that this possible disadvantage would be outweighed by their immediacy and relevance. Because our resources are typical of most mathematics departments, it was envisaged that no more than 1 BBC computer (without printer) would be available during each lesson. We are used to directing small groups of pupils to a range of activities within a topic, and the use of the computer would be an extension of this, widening the range of options and experiences available to the students.

A number of programs have now been produced encompassing a variety of learning activities. Some programs, for example, provide straightforward testing or practicing of skills, whilst others allow opportunities for the checking of results or the generating of data. We have always appreciated the need for regular review of school-produced materials, and this is happening with the programs. Questions of standards of presentation, and consistency of operation are obvious ones, but interesting matters involving teaching strategy are also arising.....for example, computers are clearly useful in producing evidence in mathematical investigations but, for some pupils, they may generate results too easily and quickly and thereby hinder meaningful understanding of the problem. We will have to give consideration to these sorts of questions if we are to make most effective use of the computer.

Programs already written or in the process of being written include the following.......

WORMS...to support an investigation. The investigation encourages pupils to study and classify the shapes produced by various "worms" which obey certain rules. The computer program allows pupils to check their drawings, or to experiment with larger "worms" than can practically be drawn.

MATRIX...practice in matrix multiplication to support a topic on matrices and transformations.

BELLS...pupils attempt to produce the required changes for a course of Plain Bob Minimus on paper and can then see and hear their results using this program; an attempt to add an extra dimension to the study of permutations and combinations.

ISBN...during a topic on Modular Arithmetic the idea of check digits is introduced. This program checks pupils answers and shows clearly the stages in the production and validation of the check digit used with ISBN. The pupils are also encouraged to wrok out how they can modify the input to this program to check the requisition numbers used for ordering school materials from the Yorkshire Purchasing Organisation.

DIAGONALS...checking and testing pupils results in an investigations of the numbers of squares cut by the diagonals of rectangualr grids.

FRUIT...a fruit machine can produce the numbers 0 to 9 on each of 3 drums. Various conditions can be devised for winning payouts. The pupils investigate the numbers of winning combinations so produced and use the program to check results or to generate results for further study.

AREA...practice at calculating the areas of shapes based on rectangles and triangles.

These programs are aimed at 3rd and 4th year pupils (actually our 1st and 2nd years since Settle High is a 13-18 mixed comprehensive). The generally informal atmosphere in lessons with an emphasis on the co-operation of pupil and teacher has given us encouraging results. Pupils have found the programs stimulating and have used them profitably whilst, at the same time, achieving a sensible balance between computer and non-computer time.

Last term we were approached by the Open University on the subject of computers and mathematics teaching and, as a result of a preliminary visit, they returned and filmed a lesson where "Worms" was being used in a topic on Modular Arithmetic. It is intended that this film and a complementary case study will form part of Open University Pack P542 "Microcomputers in the Classroom".

Our aim is that the computer will be seen in a similar light to the pocket calculator...a useful addition to the resources available to mathematics teachers broadening the range of learning activities that can be offered...in the spirit of paragraph 243 of the Cockcroft Report.

John Kitchell (Head of Maths Dept.)
Jack Upsall (in charge of Computer Studies)

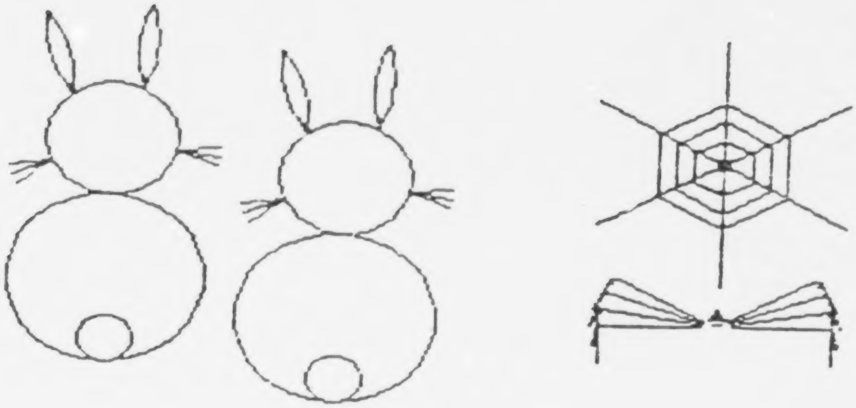
4.3 Learning Environments

Group Members:
Heather Chapman. Australia. Sharon Dugdale USA
John Hunter. UK Richard Phillips UK
Bruce Nulty. Australia Peter Samuels. Australia.
Chair/Reporter: Richard Phillips. UK

4.3.1. Discussion Points

How might different environments effect students' learning? Let us consider a very famous 'learning environment'first. Children given access to LOGO are in the words of Papert (1) 'offered a learning environment in which to flourish'. A learning environment in which they are in the 'Driving Seat'. LOGO is a very powerful example of a learning environment and an example that clearly promotes the student into mathematical thinking and exploration. We could discuss the 'primitives' that LOGO supplies the child with and consider if these are the best for promoting certain activities. Papert claims that with LOGO the child drives the machine whereas so often in other cases the machine drives the child. However, challenging this in a television interview, Weisenbaum (2) suggests that if children in several countries using LOGO all produce 'flowers and houses' perhaps the children are not operating quite so freely as has been suggested.

We can certainly look for other software that creates learning environments, indeed any program with sufficient flexibility can become a world to explore. Among the more familiar examples are adventure games and simulation programs.



Primary work with LOGO from AUCBE's Turtland

Another aspect that we need to consider carefully is an enviroment that offers children shared experiences. The following extract from 'USING THE COMPUTER TO FOSTER CREATIVE INTERACTION AMONG STUDENTS' (4) will illustrate this aspect.

INTERACTION AMONG STUDENTS

"Terminals are grouped together in the classroom in order to encourage interaction among the students. A student will often be proud of something he has done and will be eager to show it or explain it to a friend. Other times a student may turn to a classmate for help with a lesson. Generally, the materials are so motivating that the interaction among students at the terminals is centered around their work and does not take them 'off task'. A student who needs help is rarely willing to let another student take over and do the work. Usually the student will accept only the help he needs in order to become self-sufficient again.

Because tasks are generated for each student individually, the students also find that it is not helpful to copy an answer from a friend's screen. They can, however, learn ideas and strategies from watching a friend. For example, two students are working at adjacent terminals. PLATO displays some balloons tied to a number line. The task is to tell where on the number line to shoot darts to burst the balloons. When a student types a number, a dart flies across the screen and lodges in the number line at the point corresponding to the student's answer. One student, unsure of what to do, looks to see what his friend is doing. (On PLATO, as elsewhere, watching a game being played often helps one to understand the directions.) His friend has just shot a dart at 3/4 and hit a balloon. So the student turns back to his own terminal and types 3/4. But the dart does not hit a balloon, because there is no balloon at 3/4 on his number line. Clearly, copying his friend's answers does not help. So he turns to his friend's terminal again, but this time the interest is in learning how to locate numbers on the number line. After a few minutes of watching and a few more trial darts, he is able to progress through the lesson on his own.

We can produce many different learning environments through the support of creative software. A second dimension however is the hardware support to the user, in particularly one that allows student interaction.

Whatever the means used, the major objective is to give the student the opportunity to manage his own learning activity and to pursue the channels of thought that are challenging to his own developments.

LOGO
(1) Massachusetts Institute of Technology, USA
(2) Joseph Weisenbaum, Massachusetts Institute of Technology, USA

TURTLELAND
(3) Katrina Blythe and Richard Noss, AUCBE, Hatfield, Herts, England
(4) Using the Computer to foster creative interaction among students, Sharon Dugdale, Plato Education Group, University of Illinois, USA

4.3.2. Group Report

Our brief was to discuss the different learning environments which made use of a microcomputer and to consider how these would effect students' learning. This is clearly a very broad topic where a number of different factors contribute to the total environment.

The software itself can be thought of as an environment. This is most evident with systems like LOGO and with adventure games, but the idea extends naturally to smaller 'microworlds' where the software consists of simulations, games or tutorial materials.

Hardware also directly affects the learning environment. Computer networks offer different possibilities from stand-alone machines. The type of hardware purchased by a school, and its physical location, have a strong influence on the way computers are used to promote learning - and indeed, if they are used at all for this purpose.

However, there can be little doubt that the most important factor in the learning environment is still the teacher. Teachers' attitudes, perceptions, experience and knowledge of computer-assisted learning (using the term in its broadest sense) still control what really happens in a classroom, even if they may sometimes choose to delegate some of their responsibilities to the machine.

In order to establish some common points of reference, we spent a little time looking at three pieces of software: DARTS and GREENGLOBS (Sharon Dugdale, Illinois University, USA, and "L" - a mathe-magical adventure (Association of Teachers of Mathematics, UK). We also watched a number of short video recordings of teachers and children working with computers, which illustrated some of the contrasting styles of classroom organisation.

In summarising our discussion, it must be noted that we differed on a number of fundamental issues. We argued the relative merits of having a single learner at the keyboard, a small group around the keyboard, and the whole class use of microcomputers where the teacher is present all the time. We were also divided on the relative merits of author languages, where teachers largely design their own materials, and of pre-packed software, where teachers have less direct control over content. However, it is clear that there is potential in all these approaches.

We seem to be in agreement on a number of important points. The existence of computers must change what we teach in mathematics, and the use of computers in the classroom must change how we teach. We saw this pressure from new technology as a positive force in mathematics education. The immediacy of response of computers, the way they often free teachers from low-level cognitive tasks, and many other computer talents, combine to make the computer a teaching aid of formidable potential. Its motivational power seems considerable and is especially striking with children who have behaviour disorders. However, we were cautious in discussing the long-term motivational value of the computer: we were generally optimistic but were well aware we were discussing forces we did not entirely understand.

Our optimism was tempered by an awareness of a great many practical obstacles which impede the successful adoption of computer-assisted learning in mathematics. Shortages of hardware and software frustrate teachers' awareness of the possibilities, and attitudes to any innovation change very slowly - it might take a hole generation before the full benefits reach the classroom.

4.3.3. Contributed Papers

Frameworks for Consideration of Teaching and Learning with LOGO-centered Curriculum Experiences; Graham Ferres, Victoria College, Malvern AUSTRALIA.

Beware of LOGO - or who has imagination; Peter Butt, City of London School, ENGLAND.

Logo Programming within the Secondary School (11-16yrs) Mathematics Curriculum; Celia Hoyles, London Institute Education, ENGLAND.

Mathematical Strategies and Concepts Through Turtle Geometry; Helen Finlayson, University of Edinburgh, Scotland.

Using Logo to Teach Euclidean Geometry; Johnny Lott.

Frameworks for the Consideration of Teaching and Learning with Logo - centered Curriculum Experiences

Dr. Graham Ferres
Victoria College, Toorak Campus
336 Glenferrie Rd, Malvern VIC
AUSTRALIA

If a computer system can be viewed as one more aid to the learning of Mathematics, then teachers currently teaching need to be assisted to learn new knowledge skills and refine teaching skills incorporating the new aid. And the introduction of a computer system into a classroom provides an opportunity for the teacher to review what and how he/she teaches for the benefit of his/her students. In particular, in what ways might Logo - centered curriculum experiences enhance schooling? To assist in the development and review of curriculum, teaching styles and integration within student programs a series of frameworks are illustrated.

These frameworks or criteria were used as the author worked as an inservice trainer and implementation coach, with Kindergarten to Grade 3 teachers and their students focussing upon Logo - centered experiences. The teachers and students had not used computers within their curriculum program prior to the research project during the last half of the 1982-83 school year. The inservice strategies adopted were a variation of the elaborated system advocated by Joyce and Showers (1982) involving cycles of lectures, demonstrations, practice, structured feedback and coaching. The set of coaching strategies adopted during the implementation phase concerned cycles of classroom observation, structured feedback, individual curriculum planning conferences, and small group sharing reviews.

TEACHING STRATEGIES

The evidence was that every teacher implemented a curriculum unit with Logo being a means of developing computer literacy including programming principles, and problem solving and spatial skills. The transfer of the teachers' training skills to their classrooms was detailed via indicators such as the teacher's developing knowledge, the time involvement and classroom environments, and the appropriateness of the curriculum activities, including adaptations to better match the students' capabilities. The following indicators were used to chart the development of the teacher's personal knowledge of Logo programming capabilities:

- the ability to verbalise Logo and programming principles as curriculum objectives,
- a content analysis of lesson plans,
- the presence of substantial adaptions in using a prepared procedure,
- the ability to create a demonstration file incorporating student projects,
- the capacity to share knowledge and skills with other teachers, and parents, and
- the teacher's self report of skills change.

The impact of the Logo - centered curriculum units upon the instructional environments of the teachers' classroom was considerable, with for examples the teachers using computer systems to the limit of their allocation on the average on 13 days during a 25 school day reporting sequence.

A variety of learning environments were organised but a typical format was cycles of demonstration, project completion and sharing times for large groups intermixed with rostered opportunities for small groups to investigate projects. In most classrooms the computer system(s) was perceived as one work station along with several other work stations at which analogous or other topic activities were completed. This mode of organisation ensured more hands-on experience for the students, but the effect was to greatly increase the total time for which the teacher was responsible and under pressure for student learning relative to the new content and technology (i.e. instructing, sequencing, intervening and advising). One major result was that during the reporting sequence this group of teachers averaged 40 hours of class time during which they were responsible for Logo-related student learning. Thus considerable changes were being made to the content of their total class program.

Examples of effective adaptations were those involving the modification of a target game to model a randomly placed planetary system, the extension of a procedure with input capability to permit grade one students to create an effect of a multitude of tunnels, the emphasis of creating procedures for either irregular or regular or a shapes and investigating the conditions for regularity to ensure from rotation, and the incorporation of a turn command in the tail of a prepared staircase procedure so that Kindergarten students might jointly build complex patterns.

In line with previous major Logo projects at Brookline (MA) and Edinburgh (Papert et al., 1979 and Howe et al., 1980 respectively) the students worked in pairs or teams. This technique demonstrates to the students that other students may have useful knowledge and that the teacher might not be the only expert person there. The teacher became more of a planner and facilitator than an exclusive expert. Also the learning became more co-operative in nature with partnerships between pupils, and between pupil and teacher. The pupils realised that sharing of ideas and helping each other to learn was both effective and legitimate.

Borrowing a notion from the computer science and problem solving fields, the teachers also worked with the inservice trainer/coach in a variety of roles in that sometimes they acted as a USER of his files/procedures, at other times they MODIFIED his material, while elsewhere they CREATED files/procedures for the accomplishment of projects. For each category they were responsible for the classroom implementation. When working with their own students the teachers were encouraged to structure some activities with this framework in mind, so that across time their students would be USERS, MODIFIERS and CREATORS. This categorisation of teaching and learning activities has some similarities with that advocated by Bull and Tipps, who wrote in terms of the teacher as a facilitator. They devised a matrix concerning the source idea of a project (learner, facilitator) and the implementation planning and coding (learner, facilitator), so as to categorise different Logo experiences. They made the point,previously borne out by the author's research, that even in the so-called natural Logo environment in which the learner generates both the problem and the solution the teacher has considerable control over the experience by the range of original and modified primitives or tailor-made procedures supplied. The power is considerable for the teacher to devise appropriate learning experience for each learner.

TEACHING CONTENT EMPHASES

The trainer's observations and the teachers' self-reports of the principles and themes they had emphasised with their students indicated that initial foci were the general principles of input / output and control of a computer. The major emphases later were:

- planning solutions,
- how to explore new situations,
- communication with a computer,
- need for exactness of coding,
- the concept of debugging (solutions are fixable)
- the packaging of instructions into groups called procedures,
- the principle of iteration,
- the power of "building blocks" to aid the planning and completion of complex projects,
- the possibility of multiple solutions, and
- the quality of shared group learning.

In addition, most teachers devoted at least several hours of class and student time to introductory off-computer activities, focussing in particular upon ideas which modelled the turtle action on the monitor (walking, graph paper drawing, use of shape sets, and programmable cars like Big Trak). The purpose of these activities were typically to assist with planning or debugging an unexpected result (often concerned with directionality issues).

Several teachers also emphasised some of the following principles, in many cases the extra principles selected for emphasis depended markedly upon grade level:

Inverses such as Right/Left, and Forward/Back; the relationship between input and output; the notion of variability, and randomness; keyboard and listening skills; and shape geometry.

STUDENT LEARNING IN A LOGO ENVIRONMENT?

To answer this question a number of major but more detailed questions concerning the student perspective when using Logo might be considered:

1. Can students of these ages learn the Logo language? Or alternatively, what programming principles can be learnt through the use of the Logo language?
2. What is the impact upon beliefs about computers, and how one learns in general? (aids computer literacy?)
3. What Mathematics do students learn?

4. Do students become more effective problem - solvers?
5. Can students learn principles and skills from curriculum areas other than Mathematics?

This research focussed upon aspects of the first four questions, in particular concentrating upon the learning of the Logo language as a means to practice problem solving techniques, and the associated issues with respect to computer literacy. The means to these ends involved the mathematical content of shapes and angles.

The role of the students in Logo programming project activities was categorized in this ten part system:

- as users of other peoples' procedures,
- as interactive users with the basic Logo commands to create visual and other solutions,
- as users of procedures with and input,
- as modifiers of procedures,
- as creators of procedures, with the teacher's assistance,
- as creators of drawings, using others' procedures,
- as independent creators of procedures,
- as creators of drawings, using their own procedures,
- as creators of procedures which incorporate others' procedures, with a teacher's assistance, and
- as an independent creator of such super procedures, incorporationg their own or others' procedures.

The teachers and trainer considered that all had emphasised using the basic Logo commands interactively to form solutions and virtually all had emphasised the use of others' procedures, and the creation of drawings using others' procedures. Most of the teachers had encouraged projects relating to the creation of procedures with the assistance of the teacher, the independent creation of procedures, and the creation of drawings using the students' own procedures. Several of the teachers had also devised procedures with variable inputs so that their students may use such procedures. They had facilitated some students investigating how to modify their own and others' procedures, and had assisted some students develop skills in creating super procedures using their own or others' procedures.

Students in three classes (grade 2, grade 1, and kindergarten) used versions of single - key Logo so that they were able to create drawings using the same range of planning skills required by other students with the full version of Logo in combining shapes to complete complex projects with either their own or others' procedures (e.g. drawing a hexagon using an equilateral triangle and the ideas of rotation and repetition). By contrast, other students at the same levels (Kindergarten, grades 1 and 2) completed activities with the full version of the language. One group of grade one students completed most of their activities within this framework using a modified version of Logo, in which commands like F 5 were equivalent to FD 50. In this manner there was less reliance upon place value skills. These variations were selected by the teachers to best match the perceived capabilities of their students for successful experiences.

Exploration and estimation activities with the initial set of Logo commands included ones such as target activities, mazes, and tracing shapes. The difficulty level for target activities varied with placement of the target on monitor overlays, the size of the target, and the number and placement of barriers between the "turtle" and the target. Formats such as golf and baseball involving multiple targets were more difficult if the targets were solid rather than open shapes. The variability of the possible fantasies associated with the format (e.g. treasure hunt, planets, cages) was extensive. Maze activities were also adapted to "racetrack" images in several cases. Off computer estimation activities involving small informal units were also beneficial (e.g. grid spaces, ice-cream sticks, match boxes).

The necessity for exactness of instructions was illustrated by an extensive number of off-computer activities initially, emphasising the giving and/or the receiving of instructions related to a limited command set. Models included students or teachers acting as robots using any number of grids such as calendars, city road maps, numbers charts, overhead projector grids, join the dots activity sheets, and "control tower" games in which one player "talks" the other safely to a landing among the barriers.

Off-computer activities to assist with planning projects and recording results of intended designs and debugging included models such as drawing on grid paper, the use of shape attribute sets, the walking out of solutions leaving trails of masking tape or shoes or Dienes blocks or markers of any description like wooden dowels or tooth picks, and the formation and pasting of patterns using cut out paper shapes.

Several teachers prepared procedures (e.g. for a line, a circle input (e.g. C3), an arc, a staircase) so that the students could be initially a user, and then later create drawings or more complex procedures using the teacher's procedures. Other examples included the case where the students activated the drawing of a randomly placed set of planets for an individual rocketship estimation activity. These prepared procedures were several times devised with a turn command as their last command, so that groups of students working together could create a combined pattern (e.g. of stairs) which might form the basis for a series of teacher questions and student analyses.

Projects explored by students related to regular shapes, irregular shapes, the combining of shapes, the rotation of movement of shapes, the formation of towers and grids, and the filling of shapes.

A number or issues peripheral to the major computing principles and problem solution strategies became central concerns for some teachers at different stages of the implementation. For example these issues included the notion of directionality regarding left and right turns, the transfer of principles understood in horizontal off-computer activities to use on the vertical monitor screen, the giving of instructions to another object such that you must view the world from the objects' perspective, familiarity with the keyboard, and analogies to the role of the computer in the solution process.

While most younger students required some confirmatory experiences relating to directionality, some students needed continual support via shoe stickers or marbles in hands. To assist with the transfer of directionality skills to the vertical screen, activities based on stairs, chalkboards, magnetic boards, flannel graphs and bleachers, were devised to practice the vertical skills before reusing the computer system. When the emphasis became one of the student giving instructions to some other student facing them on the stairs, or to someone operating Big Trak facing them at the other end of the room or acting as "control tower personnel" then greater understandings of the turtle's view of the world on the vertical monitor screen were perceived. Special activities to assist familiarity with the keyboard were not as essential as initially expected, although several teachers of younger students color coded important keys. Several teachers use analogies such as the computer was like a pencil or a knife or a pair of scissors which needed an operator to plan its use and produce an effect. These analogies were most helpful to younger student developing the possibility of control in creating the output on the screen.

Lastly, a few teachers reflected that the following issues were of interest or some continuing concern to them with their students:

Place value understandings; the use of upper case letters in Logo; the confusion when lower grade students used two variable inputs (e.g. in creating rectangles); the frustration in "losing" the turtle behind the split text screen and apparent need for a capability to limit the students' moves to the top of the screen; the lower grades students memory loss of technical details between sessions (although teachers were often surprised concerning the amount of skill retained); and the link between this computer "language" and the students' developing language (e.g. the spacing of commands, as are words, and reading skills).

Some partial answers to these concerns involved the use of trainer/teacher devised procedures from which only single digit inputs were necessary, and/or only one input was required (e.g. the command TB 4 might create a box of constant width 20 units and height 40 units).

The problem solving strategies observed when using Logo did bear close correlation to those regarded as desirable skills by mathematical problem solving projects (Krulik and Rudnik, 1981; Schaaf, 1983). The skills often observed included the analysis of the problem into manageable sub-parts, sometimes by an analysis of a visual pattern into its repetitive segments; the solution of the sub-parts by cycles of prediction, planning, checking and refining (debugging); and when complete, the consideration of the alternative solutions often possible, and suggestions for new projects that have arisen from the first one.

CONCLUSION:

The development of the series of frameworks enabled a more purposeful evaluation of issues such as the transfer of the teachers' training skills to their classroom teaching situations, the range of teaching content emphases (programming principles, computer literacy, Logo, problem solving), the variety of teaching roles (expert/facilitator), and the style of teaching/learning roles (user, modifier, creator; and project generator and/or solver).

REFERENCES:

B Joyce & B Showers. The coaching of teaching. Educational Leadership, 1982, 40(1), 4-10.

S Papert, D Watt, A Di Sessa & S Weir. Final Report of the Bookline Logo Project (Volumes 2 and 3). Cambridge, MA: M.I.T., September, 1979.

J Howe. Developmental stages in learning to program. In Hoffman and Klix (Eds.), Cognition and memory: Interdisciplinary research of human memory activities. Amsterdam, North-Holland, 1980.

G Bull & S Tipps. Problem spaces in a project-orientated Logo enviroment. The computing Teacher, 1983-84, 11(5), 54-7.

S Krulik & J Rudnick. Problem Solving: A handbook for teachers. Boston, MA: Allyn and Bacon, 1981.

O Schaaf (Dir.). Lane County Mathematics Project: Problem Solving in Mathematics. Eugene, OR: Lane County Educational Services District, 1983.

Beware of LOGO - or who has the imagination?

P J Butt
City of London School
LONDON ENGLAND

Three days before giving a demonstration concerning the use of micro-computers in the teaching of statistics I had to rewrite my introduction. Originally having written 'I have never considered using a micro in the small amount of statistics that we teach prior to A level', I finished up by admitting that I do not have the imagination to use a micro with junior classes.

The suggestions and ideas which follow are not 'inspired', but when one considers that the computer programs that sparked off the ideas could have been written by a seven year old, then what we teach and what we encourage to be investigated could be greatly expanded.

The fact that young children using their own algorithms will be able to 'see randomness' and so absorb a fundamental fact, that there is variation, will surely make the underlying principles of statistical inference easier to comprehend in later life.

The events which caused this chain of thought, started after writing a procedure to draw a square using the turtle graphics subset of LOGO; see procedure SQUARE. Discovering the SET X and SET Y commands which allows the turtle to be positioned anywhere on the screen, the procedure RANDOMSQR WAS WRITTEN. This placed the turtle at a randomly selected point before drawing a square of size 20. Then followed the idea, REPEAT 20 RANDOMSQR which resulted in fig 1.

At this point a colleague commented, "It is interesting how much space is not covered". Then followed a flood of other questions and possible investigations.

How many 'clusters' of squares are there?
What is the 'size' of a cluster?
What is the greatest number of squares that enclose a particular point? Is this a simulation of 'ERNIE' and/or 'bad luck' - where some people seem to win often and others never - where some people suffer from one thing after another?

When considering areas:-
Are there as many squares in the left and righthand sides of the screen? Is this the same as the top and bottom halves? How does one cope with squares that cross the dividing line?

"If you draw a grid of evenly placed horizontal and vertical lines then you have simulated the fairground problem of rolling coins". So procedure GRID which uses HORIZONTALS and VERTICALS was written. See fig 2, the grid lines were drawn in a different colour to the squares.

At the meeting a member of the audience asked if it was easy to draw circles using LOGO? It is. "You could simulate raindrops falling on a paving stone, which I always find fascinating to watch".

How many raindrops are required to completely wet a paving stone?

If one were to write a computer program to enable a serious statistical investigation on any of the ideas mentioned then PASCAL or BCPL would be sensible languages to use, but a seven year old would not use those languages. Whereas it is claimed that a four year old can have a 'LOGO-world' and it is this 'world' which has given me, a forty year old, another model of randomness.

The following procedures are written in Research Machines LOGO

```
SQUARE 'LENGTH
REPEAT 4 FORWARD:  LENGTH AND RIGHT 90

RANDOMSQR
SETX PICK 280
SETY ADD PICK 160 20
SQUARE 20

HORIZONTALS
SETH 0
MAKE 'N 0
DO SETX 0 AND SETY:  N AND FORWARD 300 AND MAKE 'N ADD:  N 30 UNTIL
GREATERQ:  N 180

VERTICALS
SETH 90
MAKE 'N 0
DO SETY 0 AND SETX:  N AND FORWARD 180 AND MAKE 'N ADD:  N 30 UNTIL
GREATERQ:  N 300
```

GRID
HORIZONTALS
VERTICALS

(version 3 dated 29-11-83 Peter Smith's improvement to procedure listings).

Fig 1.

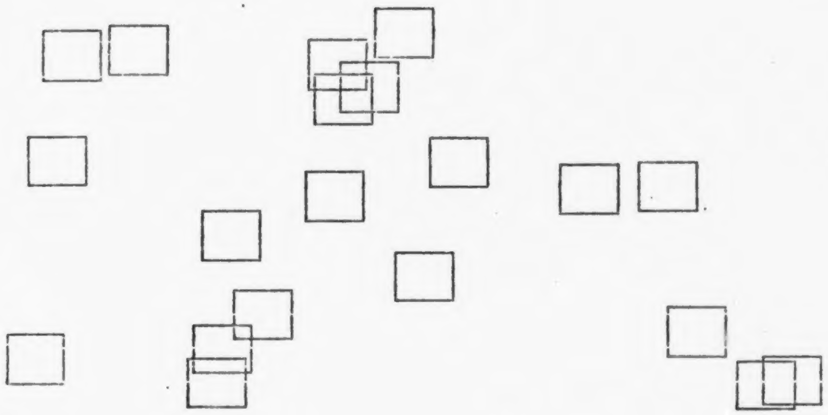
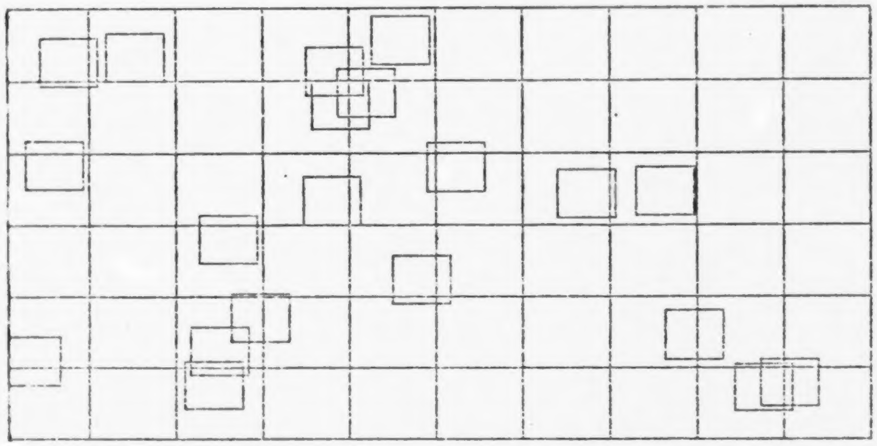


Fig 2.



Logo Programming Within the Secondary School (11-16 years) Mathematics Curriculum

Celia Hoyles

INTRODUCTION

Inevitably microcomputers will be used in the schools of the future but quite how they should be used is not at all obvious at present. Many teachers are uncertain about the role of the microcomputer in their teaching and are worried about how to organise programming activities in thier classrooms. The unthinking use of the computer or its use in situations where other materials might be more appropriate could lead to the loss of much of the potential of the new technology for learning. The use of the microcomputer in the current school curriculum is thus an important area for research.

The relationship of mathematics to the microcomputer is unique in "the extent to which the subject matter is intimately related to the operation and working of the machine itself and in the extent to which pupils can learn the subject by writing programs." (Pendley Manor paper, 1983). In addition the way the computer responds to the user by providing immediate feedback as to the outcome of any program, undoubtedly stimulates discussion and the desire to investigate problems. There is also evidence that the microcomputer enhances motivation for learning: "Pupils in large numbers are finding a joy and zest in some aspects of mathematics which they did not find before" (Fletcher, 1983).

At present the programming language most widely avaialble in schools in the United Kingdom is Basic. It is therefore most widely used in schools although there is evidence that Basic is difficult for some pupils to learn particularly at the lower end of the secondary school. The computer language used for programming undoubtedly influences the way a learner will come to approach a problem. Logo has been chosen as the appropriate programming language for the research project outlined in this paper for the following reasons:

It is a "natural" language with straightforward semantics and syntax. It is also a structured language (in contrast with most versions of Basic) which means that in Logo a complex problem may be solved in the following three stages.

- split the problem into its simpler components;
- solve each component;
- build up complete solution form all the part solutions.

These "three stages" not only make programs in Logo easier to construct and "debug" but also represent an important and more general experimental approach to mathematical problem solving. Finally, the introduction to Logo through "turtle graphics" provides an important visual dimension to programming. It is easily understood and enjoyed by a wide range of ages and abilities and yet embodies powerful ideas which can be used in other mathematical contexts or in later computer work.

Research undertaken to date into the use of Logo can be divided into two broad categories:

- the investigation into the problem solving strategies, models of thinking and mathematical processes that children develop while learning Logo in a largely unstructured manner (Feurzig and Papert, 1969; Statz, 1972; Seiman, 1980; Angleo, 1981; Watt, 1982; Noss, 1984).
- the investigation of the use of Logo as an aid to learning specific mathematical topics within a tightly structured framework (Howe, O'Shea and Plane, 1979; Hartley, 1980; Du Bouley and Howe, 1981; Du Bouley et al., DAI Paper 118(b), 1982; Du Bouley, Howe and Johnson, 1982). The former group of research projects are set within the primary school, the latter within the secondary school.

The research described here has attempted to encompass and integrate these two focusses of past endeavour.

OUTLINE OF RESEARCH TO DATE

Research, funded by the Leverhulme Trust, was started in London in September 1983. The main aim of the research is to investigate the ways Logo may be used as an aid to pupils' thinking and learning in mathematics at the secondary age level. The subsidiary aims are to:

develop materials which integrate Logo programming into the mathematics curriculum;

investigate effective teacher strategies for the Logo work;

investigate effective pupil strategies and ways of working with Logo.

During 1983/4 work has been undertaken which has aimed to investigate in particular a pupil-centered approach to learning Logo in the secondary school, within a case study methodology which has provided information on the social dimension of the classroom activity (e.g. pupil interactions and collaboration, individual differences in strategy and style, nature and extent of interaction) as well as the cognitive and affective responses of the pupils. Work has been undertaken in two first year mixed ability mathematics classrooms. In one classroom the pupils have the use of two RML 380Z computers and in the other classroom they have the use of four Commodore 64 computers (all Commodore equipment has been lent by Commodore). The pupils work in pairs and are given the freedom initially to devise their own goals and decide how they might achieve them. This ability to take responsibility for one's actions, to take resks and see what happens, to experiment and find our for oneself are all curcial elements for effective learning, that is, learning that can be used flexibly and creatively at a later date. Structured group tasks with specific learning objectives both in the areas of mathematics and problem solving, have also been used and are seen as important in building links between the Logo activity and the "standard" mathematics curriculum.

The researchers have acted as participant observers in the classroom conducting detailed case studies of fourr pairs of pupils from each school. Recordings of the pupils' Logo work and spoken language (the video recorder is connected between the computer and the monitor) have provided an extremely powerful data base and allowed the researchers to analyse in depth the progression in learning.

When and why interventions were required during the pupils' Logo activity is being at present analysed and the consequences of these interventions in terms of the pupils' motivation and learning recorded. In addition analysis is under way into the nature and extent of the collaborative work between the pupil and how this is related to the goals chosen and to the mathematical processes and the problem solving strategies adopted.

Data obtained to date indicates that some of the "powerful ideas" of Logo (e.g. modularity) do not come naturally to children and that some of the potential of the language for either the understanding of mathematical concepts or in its use as a context for mathematical processes may be missed in a completely unstructured approach (see also Leron, 1982 and Pea and Kurland, 1983). However, the experience during 1983/4 has also led the researchers to recognise the importance during the Logo work of:

some unstructured pupil-centred activity which allows pupils to explore in their own learning style and build up their confidence in and control over the language (see also Papert, 1971; Solomon, 1982 and Pask, 1976);

the role of collaboration and discussion as a means by which the pupils themselves (without a dependence on the teacher) work out a more "objective" and reflective approach to problems (see also Mugny et al., 1981; Hoyles, 1984) and stimulate each other towards more creative and investigative attitudes (see also Johnson and Johnson, 1980; Devries, 1978).

In addition there is evidence of the frequent spontaneous occurrence of mathematical ideas during the Logo activity. These ideas do not however appear to become immediately "connected" to the pupils' network of mathematical understandings which have been acquired during their "normal" mathematics work. Ample evidence has been collected of the context-bound nature of mathematical understandings and how more structural links between Logo and non-Logo contexts need to be developed.

RESEARCH FROM SEPTEMBER 1984

It is proposed that the work will continue during 1984/7 and will be divided into two parts which will take place simultaneously:

A continuation of the case study work in one school to follow on the study of children using Logo in their mathematics classes into their second, third and fourth years of schooling (up to 15 years of age).

An extension of the case study research into a wider network of ten schools to note the similarities and differences in response to the Logo activities in different classrooms where the responsibility for the Logo work is with the mathematics teachers rather than with the researchers.

The researchers intend to:

devise an approach to the learning of Logo within a flexible structure of prescribed structured tasks and investigations and unstructured pupil activity;

devise activities which use Logo programs to investigate mathematical topics or which develop links between the Logo and non-Logo contexts of mathematical ideas;

use a variety of pupil groupings, and in particular to use collaborative group work as a means to stimulate development in programming skills or grasping a particular mathematical concept;

collect data on the ways pupils approach the Logo work, noting similarities and differences in style (with particular reference to gender differences);

ascertain the reactions of teacher and pupil to the Logo activities;

collect pupil data to ascertain their attitude to the Logo work and to measure their attainment in a range of mathematical topics;

synthesise the classroom data with information obtained from teacher and pupils.

For further information of the research please contact:

Professor Celia Hoyles
Department of Mathematics, Statistics and Computing
University of London Institute of Education
20 Bedford Way
London WC1H OAL
England

REFERENCES

J d'Angelo (1981) Lamplighter LOGO Project, ADCIS Proceedings 297 (Abstract only) (A brief account of work at the Lamplighter School, Dallas).

B du Bouley, J Howe & K Johsnon (1982) Relearning Mathematics through LOGO DAI Research Paper No. 178 (Dept. Artificial Intelligence, University of Edinburgh).

B du Bouley & J Howe (1981) Relearning Mathematics through LOGO: Helping student teachers who don't understand mathematics in J Howe & P Ross (eds.) Microcomputers in Secondary Education, Kogan, pp. 69-81.

J S Brunner (1964) The Course of Cognitive Growth, Am. Psych., 19. 1-15.

Cockcroft, et al., (1982) Mathematics Counts, HMSO.

E De Corte, LOGO and Learning to Think, (a paper presented at the Conference of British LOGO User Group. Loughborough University of Technology, 2-4 September 1983).

W Feurzig, S Papert M Bloom R Grant & C Solomon (1969) Programming Languages as a Conceptual Framework for Teaching Mathematics, Report 1889, Bolt Beranek & Newman, Cambridge, Mass.

T J Fletcher, Microcomputers and Mathematics in Schools - a discussion paper by HMI Department of Education & Science.

T J Fletcher, (Ed.) (1983) Pendley Manor Paper: Paper produced by the Mathematics and Microcomputers Conference held at Pendley Manor, 3-6 May 1983).

J R Hartley, Using the Computer to Study and Assist the Learning of Mathematics (Occasional Paper No. 2, included in Proceedings of BSPLM Conference, Nottingham University, 3-4 January 1980).

J A M Howe, P M Ross, K Johnson, F Plane & R Inglis (1982) DAI Working Paper No. 188(b).

J Howe, T O'Shea & F Plane (1980) Teaching Mathematics through LOGO Programming: an evaluation study in E Tagg & R Lewis (Eds) Computer Assisted Learning: scope, prospects and limits, north-Holland.

C Hoyles (1982) The Pupil's View of Mathematics Learning, Ed. Stud. Math., 13, 4.

C Hoyles (1984) What is the Point of Group Discussion in Mathematics? (A theoretical exploration of the nature of a discussion and the framework for group work).

D Johnson & R Johnson (1980) The Instructional Use of Co-operative Competitive and Individualistic Goal Structures (in H Walberg (Ed.) Educational Enviroments and Effects, McCuthen).

U Leron (1983) Some Problems in Children Learning LOGO, Proceedings of International Group for Society of Learning Mathematics, Israel.

G Mugny et al., (1981) Interpersonal Co-ordinations and Sociological Differences in the Construction of the Intellect (in G Stevenson & G B Davis (Eds) Applied Social Psychology, Vol. 1, Wiley).

R Noss (1983) Doing Maths While Learning LOGO, Maths Teaching, 104.

R Noss (1984) Children Learning LOGO Programming, Interim Report No 2, Children LOGO Project.

R Noss (1984) Progress Report.

S Papert (1980) Mindstorms: Children, Computers and Powerful Ideas, Harvester Press.

S Papert (1971) Teaching Children to be Mathematicians vs. Teaching about Mathematics, MIT AI Memo No. 249 LOGO Memo No. 4 (July).

G Pask (1971) Styles and Strategies of Learning, Br. J Ed. Psych., 46, 128-48.

R D Pea & D M Kurland (1983) On the Cognitive and Educational Benefits of Teaching Children Programming: A Critical Look (in New Ideas in Psychology, 1983 1 (3) Elmsford NY; Pergamon Press).

M Sage & David J Smith (1983) Microcomputers in education: A Framework for research (A consultative document for the SSRC Education and Human Development Committee) SSRC Press.

A Schmitt (1975) Simulative Transfer of Mathematical Concepts by Interactive Programming (in Computers and Education, O Lecrame & R Lewis (Eds), IFIP World Conference on Computers in Education, 2nd, Marseiles).

R Seidman (1980) The Effects of Learning the LOGO Computer Programming Language on Conditional Reasoning in School Children; unpublished PhD Thesis, Syracuse University, New York.

T O'Shea (1978) Artificial Intelligence and Computer-Based Education Computer Education (30) 25-8.

C Solomon (1982) Introducing LOGO to Children, Byte (7,8) 196-208 (The Planner,Micro-Explorer and Macro-Explorer).

J Statz (1973) Syracuse University LOGO Projects, Final Report, Syracuse University, New York

D de Vries & R Slavin (1978) Team Games - Tournaments: A Research Review, J. Research Level in Education, 12. 28-38.

D Watt (1982) LOGO in the Schools, Byte (7, 8) 116-34 (Summary of Edinburgh University, Brookline, Lamplighter and New York Public Schools Projects).

M Witkin, P Oltman, E Raskin & S Karp (1971) Manual for the Embedded Figures Test.
H A W itkin (1962) Psychological Differentiation: Studies of Development, New York, Wiley.

Mathematical Stragtegies and Concepts Through Turtle Geometry

Helen Finlayson
Department of Artificial Intelligence
University of Edinburgh
SCOTLAND

ABSTRACT

This paper dscusses the reasons for the relatively poor research support given to the claims of LOGO programming as a means of developing problem solving and mathematical skills. It points out the need to be specific about the aims and direction of the programming experience. A study using Turtle Geometry with upper primary school children is then described. Analysis of the children's work is used to identify the mathematical concepts and strategies which can be developed through this approach. Finally, a second study to test the transfer of these mathematical skills to normal school mathematics is outlined.

THE CONTRIBUTION OF LOGO PROGRAMMING TO MATHEMATICAL EDUCATON

LOGO is one of the best purpose-built programming languages for mathematics education. It was developed to provide a modelling tool, suitable for use by children, to explore mathematical ideas, (Feurzeig 1969). A lot of claims have been made for it as an educational medium, but there is still not a lot of strong supporting evidence for the benefits which come from using it. Most people who have been involved with pupils learning LOGO have become very enthusiastic, so it is interesting to see why there is a lack of supporting evidence.

The claims for LOGO fall into two main categories; the contriubution towareds problem solving, and the contribution towards mathematical education.

Problem Solving

IN 1957 Polya produced a book on problem solving, outlining some general heuristics for approaching a novel problem (Polya 1957). These included the search for a similar problem to suggest an approach, the breaking down of the problems into smaller parts and finding the solution to each of these, then putting the parts together to form the whole and testing and ammending it. Writing a computer program in a well structured language is quite closely analogous to this process, and so it was considered that practise in programming would improve problem solving skills (Feurzeig 1969).

LOGO, being a particularly user-friendly, well structured language which could be used by children, was then claimed to contribute to general problem solving skills, and this hypothesis was tested in several early studies. Statz used a range of novel problems to test the hypothesis, such as the Tower of Hanoi and anagrams, but found no significant improvement in these after LOGO experience, (Statz 1972).

Since this time, models of human problem solving have undergone some changes. The earlier models stressed the strategies and procedures followed. The influential Newell and Simon model (1972) saw the task environment as encoded into problem space. Within the problem space are a set of knowledge states which can be transformed into others. Reaching a solution depends on a successful search of the problem space. The main strategies involved are planning, decomposition of the problem into subgoals, and means-ends analysis, attempting to reduce the difference between the knowledge state and the goal state. Later models (Simon and Simon 1978) recognized the importance of a specific knowledge base. The difference between a novice and an expert performing a problem solving task lies not only in the different strategies employed, but in the choice and order in which the strategies are tried. The expert is able to limit his search of the problem space because of his underlying knowledge of the problem area. The novice, following the best general problem solving heuristics available, is unlikely to find the most efficient strategy in an area with which he is unfamiliar.

Greeno (1978), looking at school geometry problems considered three categories of knowledge:

Perceptual pattern recognition

Knowledge of propositions for making inferences, i.e. specific knowledge of geometry

Strategic knowledge, involving planning and setting subgoals

From this perspective it seems unrealistic to expect programming in LOGO to assist in the solution of anagrams, as there are no shared specific skills between them, the hypothesis that programming assists in the development of general problem solving strategies may still stand, but it cannot be tested in an area where the student has no underlying knowledge base, and thus is very difficult to assess.

Mathematical learning

Other claims for LOGO are made, as a modelling tool for mathematical processes. The hidden rigour of mathematics is made explicit in programming statements (Schmitt 1975), and partially understood algorithms are necessarily clarified as they are modelled (Howe 1984). Studies investigating the effects of mathematics learning through using LOGO programming have been more realistic and shown some qualified success. Milner (1973) used LOGO programming to teach the concept of a variable, and Hartley (1980) reported a study of teaching fractions using LOGO, both with success. Papert (1979) identified some mathematical concepts encountered in turtle geometry, and showed that children improved in their ability to estimate angles. In each case the investigation was into a limited and relevant area of mathematics. Howe used LOGO within a set mathematics syllabus and tested for improvements in general mathematical performance. In one case, where the LOGO work had been substantial there were clear indications of improvements (Howe 1980), but in the other study carried out in a classroom, children spent much less time working on the computers, and the results were less clear cut (Howe 1984). In this case only a few mathematical topics were investigated through LOGO, but the assessment was made over the whole syllabus.

The particular programming context in which LOGO is learned must effect the outcome of the learning. Programming directed towards mathematical problem solving is more likely to contribute to problem solving ability than if children are just learning to program in an unstructured environment (Pea 1983, Pea and Kurland 1983). The aim of the teaching process, and how this is perceived by the learners, is also very important. Any practising teacher would recognize the need to train children for a particular approach, whether it is individual investigation, the use of worksheets or collaborative group work, but such practical considerations have tended to be ignored in research studies. If the aim is to teach problem solving skills in a mathematical context, then this should be clear to the students, and its success should be assessed in the same context of mathematical problem solving.

RESEARCH WITH UPPER PRIMARY SCHOOL CHILDREN

This investigation set out to identify the particular mathematical concepts and strategies which could be developed through turtle geometry, by upper primary school children, and to test for the transfer of these concepts and strategies to normal school mathematics.

The study was in two parts. The first phase was spent in the identification of the concepts and strategies which could be developed through turtle geometry, and the development of teaching materials directed towards this learning. This involved a preliminary study with a class of 9 year olds and a class of 11 year olds, working with the floor turtle, and extended work with the same 11 year olds using TI99/4a microcomputers. The second phase, directed towards the evaluation of the mathematical learning, is currently in progress, with a second class of 11 year olds, using the TI99/4a computers. This paper reports the findings of the first part of the study, the identification of mathematical concepts and strategies in turtle geometry.

THE APPROACH

As an introduction to programming notions, a floor turtle driven through a button box was used.

Layout of the Button Box

BUILD	FORWARD	LEFT	7	8	9
END	BACK	RIGHT	4	5	6
HOOT	LIFT	DROP	1	2	3
REPEAT	A	B	C	0	STOP
					COMMAND

This is a very simple system, but it covers the programming notions of

direct body centred commands

repeat of one command

building up to 3 simple procedures at any one time on the 3 buttons labelled A B and C

use of subprocedures

repeat of procedures

recursion

The turtle draws on a board on the floor, and this is large and easy to follow, so children can act out turtle moves.

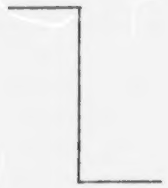
A guided discovery learning approach was used, introducing new programming or mathematical ideas gradually, one at a time, and workcards were prepared for the children, suggesting things they might like to try to do. It was considered important for the children to be familiar with direct commands before they tried building procedures, and to attempt things with which they would be successful, at least initially.

Children came out of class to work on the floor turtle in groups of 4, so that each child had one hour each week. This continued for one term. They kept notes and drawings and prepared what they were going to do while waiting for a turn on the button box. Their turtle work was monitored, and at the end of the term they each did a short turtle performance test.

RESULTS

The monitoring of the turtle work of individual children revealed considerable differences in understanding between the 9 year olds and the 11 year olds, although they had spent a similar amount of time using the turtle, and showed no great differences in their programming ability at this level. A total of 54 children were tested and almost every one could operate the button box and build and run procedures. Eleven of the older children spontaneously built procedures in the course of creating patterns, compared with 4 of the younger ones, but the real differences lay in their mathematical understanding.

On the turtle performance test given at the end of the experimental period, the 9 year old (primary 5) children were asked to draw a simple shape



and to build a procedure to draw a square of triangle, and then to make any pattern using the shape they had built. They were also asked:-

1) What determines the size of the drawing? (this was to differentiate the forward input from the angle input).
2) What difference would it make if all the left and right turns were changed over in a turtle drawing? (the mirror image of the original shape would be produced).
3) They were asked about the unique relationship between an angle and the shape it creates, with respect to the drawing they had and actually done.

The Primary 7 children (11 year olds) did a similar performance test. They were actually tested before the primary 5 children and were not given the questions. Instead, after doing the same drawing tasks, they were asked to build a 'V' and to repeat it three times, and to draw a 10 or 12 sided polygon, with the help of a sheet showing the commands for 3, 4 and 5 sided polygons.

PRIMARY 5 TURTLE PERFORMANCE

Twenty six children were involved in this part of the study and although they could operate the button box and build and run procedures, many of them found the mathematical thinking underlying the turtle work quite difficult. In the early sessions it was quite common for children to confuse the inputs of the FORWARD and RIGHT (or LEFT) commands, not recognizing the completely different effects. Five of the children were still unable to differentiate these effects at the end of term.

Seven children were not clear about the mirror image effects of changing over left and right turns. The most common difficulty seemed to be caused by overgeneralizing, or not paying attention to the orientation of the drawing, in that they would predict that an identical shape would be drawn, not a mirror image.

Many more of these younger children did not recognize that the commands to draw one particular shape could not be used to draw any other. This has been labelled the conservation of shape. One time a boy drew a square using the commands alternating a forware amount with LEFT 90 four times. He then attempted to draw a staircase using identical commands.

In another incident two reasonably bright boys were discussing their work as they drew polygons. They had both drawn a square using REPEAT 4 times and the angle 90, and a triangle using REPEAT 3 and the angle 120. The first boy said he could draw a 6 sided figure, and used REPEAT 6 and the angle 60 to do so. The other, not to be outdone, said he could draw a 10 sided figure, and chose to use REPEAT 10 and the angle 90.

square	REPEAT 4	FORWARD 50	RIGHT
triangle	REPEAT 3	FORWARD 50	RIGHT 120
hexagon	REPEAT 6	FORWARD 50	RIGHT 60
decagon	REPEAT 10	FORWARD 50	RIGHT ?

Both were surprised when a square was drawn, and discussing the error decided that he should have used 60 degrees instead, because it worked for the hexagon. This was difficult in that the problem had two dimensions. To a certain extent they were distracted by the REPEAT command, which they had correctly realized was related to the number of sides, but had they understood the conservation of shape, they would have seen that the two angles they should not have used were 90 and 60 degrees, because they already knew that 90 draws a square and 60 draws a hexagon, and therefore could not draw any other shape!

In the final turtle performance test eleven primary 5 children fell down on the conservation of shape, and having agreed that a square was drawn by repeating the two commands, FORWARD some quantity and RIGHT 90, then confidently stated that a triangle could also be drawn using only these two same commands. Some also said that squares could be drawn using different angles, such as 20 or 60 degrees. In the interview care was taken to check that his was a genuine lack of understanding, not just poor explanation of a deeper understanding.

PERFROMANCE OF 11 YEAR OLDS ON THE FLOOR TURTLE

There was no evidence in the primary 7 turtle work of children confusing the angle and forward inputs, or of not understanding the conservation of shape, though this was not tested. Quite often if a child had turned left instead of right he would correct this mistake in two stages

```
LEFT 90
RIGHT 90
RIGHT 90
```

and when using estimation to find an angle would use several turns in the same direction

```
RIGHT 90
RIGHT 20
RIGHT 10
```

but on repeating the drawing would usually simplify these commands, showing a good understanding of equivalence values.

All these children, except one with particular spatial difficulties, had a good understanding of mirror images. This one boy had problems differentiating left and right turns, and in the turtle performance test attempted to draw a closed shape, a triangle, using alternating left and right commands. He did not recognize that simple closed shapes are produced by turning consistently in one direction.

Only four of the primary 7 pupils were not successful in the earlier part of the test. These were the above mentioned boy, and three other children who had difficulties with finding the right magnitude of angles to use.

IDENTIFICATION OF MATHEMATICAL CONCEPTS

All the children involved in the study already understood linear displacement and had little difficulty in grasping the additive and inverse properties characterized by

```
FORWARD 20 +  FORWARD 20 ——>FORWARD 40
FORWARD 40 +  BACK 20    ——>FORWARD 20
```

Most geometric concepts were completely new to the 9 year olds. From the errors they made it was possible to trace the development of the concepts of angle and shape.

CONCEPT OF ANGLE

The first requirement for the development of this concept was the ability to differentiate between the input to FORWARD (or BACK) command and the input to RIGHT (or LEFT) command. The former causes a change in the position of the turtle with respect to the board, whereas the latter changes the turtle's orientation only. Children who found difficulty differentiating the two would typically make errors when reducing the size of a figure. The tendency was to reduce the angle input as well as the forward one. This was a very common early bug, but most children later avoided it.

Most children then went through the stage of using fixed size angles (usually 90 degrees, as this was the example given to them) with an intuitive recognition of the inverse relationship between LEFT 90 and RIGHT 90. Very few children remained at this stage, but on the final performance task one boy, asked to draw a triangle using 120 degrees, used the commands.

```
FORWARD 120
RIGHT 90
```

ILLUSTRATING THIS PROBLEM

The next stage was the recognition of angles of varying amounts, without fully accepting the conservation of angle, or its additive properties. Children would use apparently random inputs when drawing a regular figure, failing to recognize identical angles.

The conservation of angle is then developed. This is the recognition that a given input to a RIGHT (or LEFT) command will always produce the same amount of turning. The inverse of this is that the same input to a LEFT (or RIGHT) command will produce the same amount of turning in the opposite direction.

CONCEPT OF SHAPE

In the limited context of working with the button box, children spent a lot of time drawing regular shapes. This was done by building a two line procedure;

```
BUILD PROCEDURE A
FORWARD 50
RIGHT (some angle)
END
```

and repeating this procedure until it formed a closed shape. From this experience using lots of different angles, the concept of shape is developed. The simple shape rule can be stated that "For any regular figure the shape is fundamentally determined by the angles used in it." In the above example, the input to the RIGHT command determines the shape of the figure, whereas the input to the FORWARD command determines its size. This can be extended to the idea of "rectangularity" involving the use of only 90 degrees.

Other general rules can be developed from this base:-

1) Simple closed figures involve turns all in the same direction.
2) Mirror images of figures are produced by reversing the left and right commands.
3) The general shape rule can be extended from the simple shape rule. Any figure can be defined by a unique ordered set of FORWARD and RIGHT commands.
4) The inverse of this rule also holds. If one set of commands produces a particular figure, it cannot draw any other figure. This has been labelled the conservation of shape.

MATHEMATICAL STRATEGIES

The development of particular concepts also involves the use of mathematical strategies such as generalization. It was found during the course of the turtle geometry work that three particular mathematical strategies were required. These were generalization, reversible thinking and information gathering. These are all important elements in mathematical problem solving which can be developed through investigative work of this nature.

GENERALIZATION

This can be taken at three levels

a) extending an idea from one instance to another;
b) recognizing and extending a pattern, informally;
c) developing a general law or expectation.

REVERSIBILITY

This has three levels

a) recognition of the inverse of a simple operation (unary +4 inverse -4)
b) recognition of a reversible process and its use, (binary or complex 26 + 4 = 30 reverse 30 - 4 = 26.) This involves having a general idea of the relations within an additive or multiplicative relationship.
c recognition of the reversibility of interactions between two or more variables. e.g. the size of a circle depends on both the forward and the angle inputs. A smaller circle can drawn by either decreasing the forward, or increasing the angle input.

INFORMATION GATHERING

This involves discrimination of relevant information from irrelevant, and using all the information available, including that generated from erroneous trial solutions, (intelligent trial and error, or successive approximations). This is an essential ability for problem solving, but is perhaps less involved in concept formation than the other two strategies.

DEVELOPMENT OF STRATEGIES

There were many instances of children using simple generalizations, but none of the 9 year old children were able to state general rules. One boy recognized that a 10 point star required a two line procedure to be repeated 10 times. He saw significance in this but made no attempt to test its generality. In investigating polygons some children were able to recognize that there was a pattern in the relations between the number of sides and the angle turned. They generalized the rule that the larger the number of sides the smaller the angle, and from this they were able to make successive approximations to find the required angle for a 5 sided polygon. Two children also recognized the halving and doubling effect relating the 4 and 8 sided figures and their angles and the 3 and 6 sided ones. They extended this to find the angle for a 10 sided polygon by halving that required for a pentagon. They did not recognize the significance of 360 degrees or formalize the rule.

Problems requiring reversible thinking were significantly harder for the younger children. The investigation of circles of different sizes required both reversible thinking and attention to at least two variables. Successful investigation also necessitated the information gathering technique of control of variables. The children again built two line procedures and repeated them as many times as required to draw a closed figure. Any polygon with 20 or more sides, when drawn by the floor turtle,was sufficiently circular to be considered, and children soon learned that a reasonably small angle was required. The relation between the forward and the angle inputs proved to be more difficult. Some chidlren, recognizing that the forward input generally determines size drew a circle with the commands

```
    FORWARD 1       RIGHT 1
```

but then demonstrated their lack of reversible thinking as they stated that it was not possible to draw a smaller circle because the forward input could not be futher reduced. Others tried to draw a circle twice as big, using the commands

```
    FORWARD 2       RIGHT 2
```

and were amazed when it turned out to be the same size.

Four of the children used the technique of holding one of the inputs constant while varying the other one, and in this way were able to recognize that the size of the circle drawn was inversely related to the size of the angle.

One particular boy showed good ability to make use of infomration and generlize when he came across a square drawn using the angle of 270 degrees. He knew that squares could be drawn using 90 degrees, and perceived the relationship

$$90 \times 3 = 270$$

He then tested the hypothesis that any multiple of 90 could be used to draw a square, and tried out 180 degrees. When this drew a straight line he abandoned the investigation.

With the older children the polygon question on the performance test was to assess their abiltiy in finding underlying rules. They had not been taught the 'polygon rule', but some of the quicker groups had spent some time playing with polygons. Thirteen of the twenty eight pupils were able to draw the required polygon, demonstrating their ability to generalize, and eight of them did so by building a two line procedure for one side and repeating it.

CONCLUSION

The mathematical concepts of angle and shape were developed through work with turtle geometry, but the time needed for children to understand these concepts varied with age. The 11 year old children already came to the LOGO work with some understanding of angle, and so spent less time assimilating the concept within the turtle geometry context.

More interestingly, both age groups could use this investigative approach to develop mathematical strategies of generalization, reversible thinking and information gathering. The spatial pattern recongition content of Turtle geometry is particularly suitable for developing such strategies.

Though there is little positive evidence of the transfer of general problem solving skills for LOGO experience, it seem reasonable to expect particualr mathematical strategies to transfer from turtle geomtry to other mathematical contexts. It could be argued that learning to look for patterns and making and testing generalizations is the essence of mathematics, though this is ofhten hidden in the details of paritcular tasks.

BEST COPY AVAILABLE

A continuation of this study is currently underway. This is aimed at developing these mathematical problem solving strategies and the concepts of angle, shape and variable, through the analogy of inputs to procedures in the more advanced programming work. Tests will begin to asses the transferability of these concepts and strategies to normal school mathematics.

REFERENCES

W Feurzieg, S Papert, M Bloom, R Grant and C Solomon (1969): "Programing languages as a conceptual framework for teaching mathematics" BBN Report No. 1889. Cambridge Mass.

J G Greeno (1978): "A Study of Problem Solving" in " Advances in Instructional Psychology" Vol 1 Ed. R Glaser. Lawrence Erlbaum Associates. New Jersey.

J R Hartley (1980): "Using the computer to study and assist the learning of mathematics". Proceedings of the British Society for the Psychology of Learning mathematics Conference, Nottingham University.

J A M Howe, P Ross, K Johnson and R Inlgis (1984): "Model Building, Mathematics and LOGO" in Yazdani (ed) "New Horizons in Educational Computing". Ellis Horwood Limited. Chichester.

S Milner (1973): 'The effects of computer programming on performance in mathematics.' ERIC report EDo 76391. Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans.

A Newall & H A Simon (1972): "Human Problem Solving" Englewood Cliffs, New Jersey: Prentice Hall.

S Papert, H Abelson, J Banberger, A diSessa, E Hildreth, D Watt & S Weir (1979): Final Report of the Brookline LOGO Project. MIT Memo No 545, LOGO Memo No 53, Artificial Intelligence Labratory, M.I.T.

R D Pea (1983): "LOGO programming and Problem Solving". Bank Street College, Centre for Children and Technology. Technical Report 12.

R D Pea & D M Kurland (1983): "On the Cognitive Effects of Learning Computer Programming." in "New Ideas in Psychology" 1983 1 (3) Pergamon Press. Bank Street CCT TR 9.

G Polya (1957): "How to Solve it" New York: Doubleday Anchor Books.

A Schmitt (1975): "Simulative Transfer of Mathematical Concepts by Interactive Programming" in Lecarme and Lewis (ed) "Computers in Education". North Holland.

D P Simon & H A Simon (1978): "Individual Differences in Solving Physics Problems." in "Children's Thinking: What develops? "Ed. R S Siegler. Erlbaum Associates. New Jersey.

J Statz et al (1972): Syracuse University LOGO Progress Reports 1,2 and 3.

ACKNOWLEDGEMENTS

The practical help and support provided by the staff and pupils of Bruntsfield Primary School, Edinburgh, are gratefully acknowledged. This work was supported by SSCR and Texas Instruments.

Using LOGO to Teach Euclidean Geometry

Johnny Lott
University of Montana
USA

Under National Science Foundation Grant No. SER-8160728, activities were developed to teach topcis in euclidean geometry to proespective elementary teachers through the use of LOGO. For the activities, Terrapin LOGO was used on Apple II+ computers. The activities were developed during the 1981-82 academic year, were revised and tested during the 1982-83 academic year. The activities were tested with three exisitng classes consisting of 100 students in Math 133 Mathematics for Elementary Students at the Unvierstiy of Montana.

The 35 developed activities were introduced to the students in a lecture-demonstration format. The lecturer used an Apple II+ with two overhead monitors, an overhead projector, and blackboard to present the activities. In a 50 minute class period, students were shown the mathematics necessary to write a LOGO procedure, the necessary LOGO primitives, and a set of procedures to accomplish the activity. Students were then required to take the LOGO procedures, perform experiments with them, answer questions concerning them, edit them to develop variations, record the results, and turn in their work on floppy disks.

A sample demonstration activity concerned with the construction of a triangle if given two angles and an included side of the triangle follow. Previous to this experience, students had used compasses and straight edges to perform the traditional euclidean construction. As in all lecture-demonstrations students were encouraged to paritcpate in the development of the procedures. For this particular lesson, students were asked to recall the compass straight edge construction. If given two angles, marked A1 and A2, and the segment length, marked S, in Figure 1, the construciton of a triagnle with angles A1 and A2 with included side S might appear as in Figure 2.

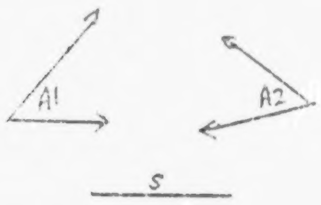


Figure 1

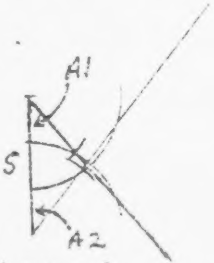


Figure 2

When asked to write a LOGO procedure to perform a similar construction, students produced the following procedure called ASA which has three inputs, :A1, :A2 and S for the two angles and inculded side respecitively.

```
TO ASA :A1 :S :A2
FD:S
RT 180-:A1
FD 100
BK 100
```

BEST COPY AVAILABLE

```
LT 180-:A1
BK:S
RT:A2
FD 100
PU
HOME
END
```

The ASA procedure, as developed produces a result very similar to the compass-straight edge construction if the two rays produced do intersect. While this is a technically correct construction, it does not use the power of LOGO to achieve the end result, and as noted may not produce a triangle at all.

With the two problems in mind, the lecturer considered the following to cnostruct a different ASA procedrue. Suppose there was a procedure written which started with the turtle at home, a point on monitor screen with coordinates (0,0), moved the turtle (an object used to accomplish LOGO drawings on the screen) backwards the length, S, and turned the turtle right the measure A2. Such a procedure would now have the turtle in position as shown in Figure 3.

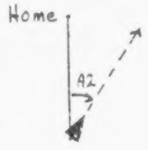


Figure 3

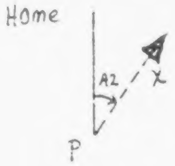


Figure 4

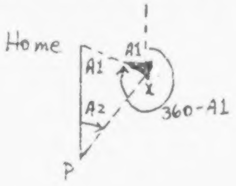


Figure 5

Now consider what is required to complete the construction. The turtle must be moved along the dotted ray in Figure 3 until it reaches a point X, as shown in Figure 4, where , if it were turned toward home, it would have heading 360-A1.

To understanding the heading, consider Figure 5. If the turtle is headed due north, it has heading 0. Turning the turtle colckwise incerases the heading. If the turtle makes one complete revolution it turns through 360°. Thus a turn toward home gives the turtle a heading of 360-A1.

To help the turtle decide when point X is reached, a CHECKHEAD procedure is needed. If CHECKHEAD were written, and could determine the location of point X, all that is requried to complete the ASA procedure is to send the turtle home, Thus, if CHECKHEAD were written the new ASA procedure is defined as follows.

```
TO ASA :A1 :S :A2
BACK :S
RIGHT :A2
CHECKHEAD
HOME
END
```

Now the CHECKHEAD procedure must be written. Consider what it must do. It must move the turtle along PX, as shown if Figure 4. It needs to stop periodically, turn the turtle toward home, determine if the turtle's heading at that point is the one desired, and if so, stop the CHECKHEAD procedure. If not, the turtle must be turned back along PX, move fowarad to another point to repeat the process.

Hence a first attempt at CHECKHEAD might be the following:

```
TO CHECKHEAD
FORWARD 1
SETHEADING TOWARD 0 0
IF HEADING = 360-:A1 STOP
SETHEADING :A2
CHECKHEAD
END
```

As described above, the turtle is moved forward 1 unit to a point before the check occurs. (The turtle could be moved forward a smaller unit if desired.) Mathematically, it should be noted that it is impossible to check every point along PX since there is an uncountably infinite number of points on a ray and only a conuntable finite number of points that the computer can check. As a result, the computer may never find the exact point X to make the turtle heading equal 360-A1. Thus, a determination must be made to determine what point is " close enough" to point X to be aceptable. If it is determined that an acceptable point is found whenever the turtle's heading toward home is within 2° of A1, then the checkhead procedure must be adjusted accordingly. To be within 2° indicates that the absolute vale of the diffenence of HEADING and (360-A1) must be less than 2°. To meet this requreiment a prcedure must be written to find the absolute value of some given value. If such a procedure called ABS is completed, then the orginal CHECKHEAD procedure must be edited as follows.

```
TO CHECKHEAD
FORWARD 1
SETHEADING TOWARD 0 0
IF ABS (HEADING-(360-:A1))  2 STOP
SETHEADING "A1
CHECKHEAD
END
```

Finally the ABS procedure must be written. This procedure must accept a value, :VALUE, and output its absolute value. If :VALUE is positive or zero, the ABS must output :VALUE; if not, it must output -:VALUE. Because of the nature of conditionals in LOGO, it ie easier to handle the cases as follows;

```
IF :VALUE  0 OUTPUT -:VALUE
OUTPUT :VALUE
```

Thus, the ABS procedure follows:

```
TO ABS :VALUE
IF :VALUE  0 OUTPUT -:VALUE
OUTPUT :VALUE
END
```

At this point, the set of procedures are complete. As activities, students are encouraged to try the following:

(1) Edit ASA to include an error message if the sum of the given angle measures, :A1 + :A2, is greater than or equal to 180.
(2) Edit ASA to include an error message if the triangle wraps on the monitor screen.
(3) Edit the ASA procedure to use smaller values of the amount the turtle moves forward and the degree error in the approximation.

5 TEACHER EDUCATION

5.1 Computers and Inservice Education: Big Problems or Big Solutions?
Richard Shumway
(reported under the ICME publications Professional Life of Teachers)

5.2 Computers and Teacher Pre-Service Training
5.2.1 Introduction
5.2.2 State of the Art in Different Countries
5.2.2.1 USA
5.2.2.2 France
5.2.2.3 Germany
5.2.2.4 Republic of China
5.2.2.5 New Zealand
5.2.2.6 Vocational Training

5.2.3 Teacher Education on Computers in an Open System
5.2.3.1 Didactical Studies of Mathematics Teachers (USA)
5.2.3.2 France
5.2.3.3 New Zealand
5.2.3.4 Consequences for Teacher Education (Germany)

5.2.4 The Effects on Mathematical Education Subject Matter, Methods and Ways of Learning and Teaching
5.2.4.1 USA
5.2.4.2 An Informatics Course for the Teaching of Mathematics France
5.2.4.3 A New Subject "Computer Education" or "Computer in Mathematical Education"?

5.2.5 Problems in School and Society
5.2.5.1 How Do We Live With a Computer and Love it Without A Heart? Republic of China
5.2.5.2 USA

BEST COPY AVAILABL

5.2 Computers and Teacher Pre-Service-Training

Group Members:
P Anderson. Austria
S Chiu. Republic of China
B Cornu. France
B Monthubert. France
Y Kimura. Japan
B Doyle. New Zealand
L Hatfield. USA
J Scott. Australia
M Tong Republic of China
F Emery. France
K D Graf. Germany
Y Terajima. Japan
C Greenbank. New Zealand

Organizers/Reporters: K D Graf. Germany and B Cornu. France

5.2.1 Introduction

There were two sessions on the "State of the Art", with reviews from different countries about computer dissemination in schools, teacher background knowledge and programs to train teachers. Each reviewer stressed one special aspect, such as "developing didactical knowledge for school mathematics teaching","the challenge of informatics to mathematical education".

One session took place as a panel, discussing questions like "what goals for which students?", "where to establish the teacher training courses?" and dealing with two short statements on "computer graphics for mathematics" and "philosophical problems with computers in education".

The final session was dedicated to the formulation of results and recommendations, covering points of agreement, points of dis-sense and suggestions for activities.

This report will document the output of this session and will back it with citations from the starting papers of some group members.

There were four main topics:

(a) The state of the art in different countries
(b) Teacher education on computers in an open system
(c) Effects on mathematical education -
subject matter, methods, way of teaching and learning
(d) Problems in schools and society

5.2.2 State of the Art in Different Countries

Great similarities exist between the different countries: Within the next five years, every school plans to be equipped with computers.

Often there are more computers than teachers able to use them.

Often there are more teachers willing to use computers than computers available.

Some special computer science courses for teachers and students have been developed.

Many uncoordinated developments have occurred.

Adequate international dissemination and reflection is required.

5.2.2.1 USA

L L Hatfield, The University of Gerogia, Athens, Georgia

Mathematics teachers at all levels of schooling are confronting the impacts and potentials of microcomputers as instructional tools. During the past two decades, a small trickle of interest and activity has swelled to a flood of concerns and efforts related to the use of computers in the classroom. Yet much of the promise of this existing, powerful technology has gone unfulfilled, as the multitude of theoretical issues and practical problems are becoming recognized.

In the recent swirl of near-frantic acquisitions of computing hardware by schools and parents, teachers have become increasingly pressured to improvise in order to "make something happen". In the United States the situation may have already reached crisis proportions in many schools, as parents and policy makers exert pressure for the implementation and usage of computers. In most cases teachers proceed without adequate planning, without didactical preparation, with few appropriate curriculum materials, and, perhaps most crucially, without thoughtfully constructed educational rationales upon which their classroom usages of computing might be based.

5.2.2.2 France

B Cornu, Université de Grenoble

The equipment of schools with computers is quickly increasing. Some schools have been able to buy some. The ministry has drafted a plan, according to which all secondary schools should be equipped wihtin the next 5 years. The standard equipment for a school generally consists of 8 microcomputers.

Languages usually available are Basic and LSE, LOGO is also available on some computers. Primary schools, when they are equipped, generally have small "home computers" or sometimes Apple II.

Except in a few cases, the schools had their equipment for a very long time; the computers are often badly and/or little used. Teachers do not know what to do with them. Software is available, but very little is really interesting; much should/could be invested in this domain.

Computer science is taught to some technical sections on secondary level, and also, as an optional subject, in some "lycées". We have to distinguish the teaching of computer science from the use of computers for the teaching of mathematics.

Besides the use of computers in classrooms many "computer clubs" have been developed.

Most of the teachers using computers are autodidacts and they have no strong backgrounds in computer science.

The IREMs (Institut de Recherche sur l'Enseingnement Mathematique) have contributed for a long time to the training of teachers; they organised many sessions for basic and continued training.

The ministry has put on special structures for the computer training of teachers for all subjects for three years. Some teachers are on paid leave for a year to attend the training; courses of shorter durations (50 or 100 h.) are also existing. But the demand for training is very strong, and at this moment, the demand can be satisfied only partially.

The pre-service training includes, up to now, no training in computer use for mathematics teachers. Teachers are hired through competitive examinations. The main exam (CAPES) includes no computer or informatics test. The "agregation", difficult competition concerning only a few teachers, will include an optional test next year: "Mathematics or informatics".

5.2.2.3 **Germany**

K -D Graf, Freie Unversität Berlin

Strong Involvement Locally - Slow Progress Globally

Training teachers how to use computers, or more general, training teachers in computer science (informatics) is a very fascinating task. This is true since very often the students are working really enthusiastically and they become very creative, too. This is demonstrated by a number of interesting textbooks written by teachers in service. Unfortunately one must also say that this fascination is only true locally, i.e. from a small group of teachers or teacher students. Looking at the problem globally, it turns out to be rather frustrating instead of fascinating. The reason is that the broad majority of teachers, including mathematics teachers or teacher students, have no experience at all with computers, not from their own time in school nor from their training, nor from their every-day-life. And this majority is not necessarily willing to tackle the problems on their own, very often due to a certain reform-weariness caused by problems with "new math" and the like.

As to teacher students, very often a heavy load from traditional subjects leaves them little time for additional activities with computers, which normally so far are not compulsory in most teacher education programs. Also, many lecturers and professors at teacher training colleges and universities still are very conservative about introducing computers in schools.

Public awareness of the problems

Things are changing, however. An understanding about information technology as a general problem of education has meanwhile reached high ranking political decision makers - essentially due to early and intense activities of teachers, who wanted to prepare their students for the computer-age in time. As a consequence, many impulses and measures are flowing back into the field of education. According to an interview with the Secretary of Education and Science, Frau Wilms, measures will be taken to introduce the teaching of basic skills in information technology to all schools and to develop teacher training as well. A working committee "school and business" will aim at activities in the same direction. Trade unions are starting with similar goals. Many mathematics teachers and professors, who are not primarily interested in computers, now view informatics as a general challenge to mathematical education on the whole and not only as a technological tool. Thus the chairman of the "Gesellschaft für Didatik der Mathematik", Professor H Winter, said in an interview: "At present the computer is the greatest challenge to math education ... Hardly any situation in the history of mathematics could be compared to this. Math education must be reconsidered and redefined as to intentions, subject matter, methods and ways of learning". On the other hand, he also has reservations: "The actions in the classroom must not be dictated by the development in computer technology, however. Not everything which can be done is reasonable from a pedagogic point of view. Possibly school has to resist and not to adapt".

5.2.2.4 Republic of China

Sou Yung Chiu, National Taiwan College of Education, Chang-Hwa City

The computer education program, if it does exist in teacher Pre-Service Training, can only manifest itself by the courses offered, personnels available, hardware employed, and rationale justified. Let us survey JNC first.

Junior Normal College

Among the nine junior normal colleges, only two of them ever offer courses in computer science, they are Provincial Taichung Junior Normal College (PTJNC) and Taipei City Junior Normal College (TCJNC).

PTJNC is the first one to take the initiative in offering a course called: Introduction to Computer, a two semester course 2 credit hours per semester. Since it is imperative to go though all the legal procedures to add a new course to the approved list of elective courses in the Curriculum Standard decreed by the Ministry of Education, PTJNC made a plea for permission to start on such a course three years ago. The course is now administered at the 4th year, electively taken by students majoring in mathematics, science, and art.

Lecture notes for course material is compiled exclusively for prospective teachers by three staff members of the college, by adapting the teaching material currently in use at serveral other well established universities: National Taiwan University, National Chung-Hsing Universtiy, and Tang-Kang University, based on their own perception of the needs of JNC students.

In the course, the following topics are included:

- Usefulness of computers, as well as the limit of their capabilities with reference to a brief account of the evolution of computers
- Structure of computers, binary number system

- BASIC language and programming
- Microcomputers
- Processing of information in chinese characters

There is, besides the course, extra curriculum activities to foster the learning and make aware the majority of the students not taking the course of computers.

A big room is furnished as computer laboratory to facilitate the learning of the course, and when necessary, to host the related extra curricular activities among students.

There are now 11 sets of microcomputers, one big set of Radio Shack. An even bigger set of Radio Shack is used for administrative work and for CMI to make statistical analysis of student's achievements to teaching staffs.

TCJNC is the second one, following PTJNC, to offer course in computer science. It makes possible, for the person formerly taking charge of the introductory course in computer at PTJNC, now transfering to TCJNC under agreement of these two colleges joint to assist TCJNC in establishing such a program of computer education.

- Rationale Justified

Partly because that in the curriculum standard of JNC issued by the Ministry of Education, there is not a computer course included in the approved list of elective courses, and partly because the newly issued tentative curriculum standards for computer education does not mention a word or even make a hint concerning about JNC, as opposed to the concern it pays to junior technology/business college, JNC feels no obligation to offer such a course and therefore does not feel pressed to appoint a person capable of the teaching charge. All the other seven JNC do not offer courses in computer or provide facilities for extra curricular activities in the computer, due to lack of appropriate personnel to assume the teaching responsibility.

- Action Taken

While it finally becomes to the Ministry of Education the necessity of computer education, that it is due to revise the curriculum standards for JNC by the addition of a compulsory course in computer at the 3rd year, the Ministry of Education has initiated a "staff-development-plan" since last year, to render in-service provision for the teachers at JNC.

Following it, each college elected one of its teaching staff to take up the one-year-study-program, with full-pay-leave, in National Institute of Industry and Technology at Taipei.

There are nine of them. Having just finished their study in computer science, these people are coming back home and will be looked upon highly by their collegues and principals, not only to teach courses, but also to start on whatever kinds of activities to supplement the learning of computers.

However, we could honestly say that: the computer education for prospective elementary teachers arrive merely at a warming up point. For the educators there are lots of issues, considerations and barriers to identify, to devise strategies to act, and to act actively.

Now, let us turn to the 4-year teacher training colleges.

Year Colleges

Each college develops its own programs based on its own interest and thus its conception of need, qualification, and modernization of educational provision. Briefly, each institute offers the following basic courses:

Introduction to computer/computer science
BASIC & Programming
FORTRAN & Programming
Microcomputers.

They are primarily for prospective teachers to prepare themselves for giving courses in schools.

However, listed in the newly revised curriculum standards for the Department of Mathematics at these teachers' colleges, none of the basic computer courses is compulsory or compulsorily elective. What can be said is that this treatment is a deliberation, and persuasion is a recommended approach to attract prospective teachers to select courses in this area.

- Rationale Justified

Restricted by the prescribed limit on the totality of 50 credit hours in compulsory courses, and holding a strong conviction that courses in computer science have already become very fashionable and will be popular among college students, the group of specialists commissioned by the Ministry of Education to develop curricular requirements for prospective high school teachers, has deliberately listed all courses for computer education as elective ones, in order to make room for more compulsory courses in fundamental mathematics.

Indeed, it is a common observation by many teachers that there are not few students who, while showing reluctance at mathematics learning, they are very enthusiastic about computers.

- Other Efforts

Efforts to use computers in the teaching of mathematics subjects, such as: freshman calculus and numerical analysis have been made to help develop the understanding of mathematical concepts, such as: limit approximation, iteration,convergence. However, the integration of computer into mathematics education is practically absent.

Effort to instruct prospective teachers in the use of computers in school, or more restictly to run a method course to prepare teachers for using computers to assist learning in school (CAI) has not been well planned. Only Chang-Hwa College offers such a course.

Teaching personnel both qualified for teaching computer and sensible for work in teacher education is almost none. Mathematics teachers in these colleges are encouraged to pursue further studies both in informatics and pedagogy.

5.2.2.5. **New Zealand**

(Chris Greenbank and Barry Doyle, Wellington Teachers College).

The first courses in Information Science were offered in New Zealand in the late sixties at the universities. By the early seventies exploratory work in Computer Education was being done in the advanced Maths Classes of the 6th and 7th Form and in some teachers' colleges. Most teacher education was, however, being handled informally through professional associations of mathematics teachers and interest groups.

By the middle seventies formal syllabus development had begun for examinable courses for some 6th courses in Numerical Mathematics and Numerical Methods. Universities had by then developed undergraduate degree courses in information science. Work in teachers' colleges was still generally informal and ad hoc. No policy on teacher education existed.

By the end of the decade formal and government funded agencies and study groups were drafting policy and were engaged in syllabus construction. Microcomputers were being aquired by some secondary schools on an ad hoc basis using parent funding.

During this period a special project directed by Neil Scott, Director of the Wellington Polytechnic School of Physics, Electronics, Technology and Engineering developed a microcomputer with architecture specifically designed for use by teachers and children.

The development and application of this microcomputer and other tools for education of the handicapped has gained international attention. This seminal work attracted the support and interest of the New Zealand Government and fostered a strong interest in the use of computers for teaching.

Teachers looked for guidance and financial assistance in the purchase of hardware and the generation of appropriate courseware. The government responded by publishing a list of the six microcomputers considered suitable for slchools. No subsidies or financial allocation was made, however, for the purchase of hardware. This response was generally greeted with dismay and disappointment by the education community. It stalled any attempt to get a unified or coherrent material system of computing in schools. Teachers responded by doing their own evaluation of hardware within strong financial constraints. Proliferation on the ad hoc purchase of diverse, cheap and often unsuitable micros resulted, thwarting the opportunity which had existed to develop, on a national scale, courseware which would be linked to the syllabus and be culturally acceptable.

In the eighties a Courseware development unit was established by the government. Its purpose is to evaluate software and guide teachers in the purchase decisions. It will also co-ordinate and facilitate some software production and distribution. Most software is, however, being generated and purchased on an ad hoc basis paralleling haradware acquisition.

There is a broad base of enthusiasm by practising teachers and a high demand for in-service training, pre-service training and educationally sound software. At Wellington Teachers College we hve a network of six Poly computers purchased two years ago and the Mathematics Department has been given an extended staffing entitlement (5) for 80 extramural students, mostly primary and some secondary classroom teachers, and 60 pre-service students who elect computer studies. No compulsory courses exist. The course developed and taught within the Mathematics Department gives strong emphasis to the applicatons of computer education in all curriculum areas. Software has been developed at the college illustrating the use of computers in teaching elements of Geography, Music, Health, Maori language etc. but the teaching of mathematics is the area with most applications and of most interest to us. Emphasis in the course, which uses BASIC as a programming language, is given to the conception and structuring of educational software. Students are expected to use exemplary programmes and to generate other examples in a selected field. The software produced in the college course by students and teachers using Poly computers and that from schools throughout New Zealand are shared freely, distributed and swapped through a 'Trading Table' centred at Wellington Teachers College. This facilitates the publication and exchange of ideas and software. It helps to force up the standard of software being generated. Similar systems of software exchange exist through user clubs for other makes of microcomputers being widely used in schools.

5.2.2.6 Vocational Training
Germany (Federal Republic)
J Loff, Osnabrück

Business Informatics and Teacher-training in Vocational Schools

Definition of terms

Business informatics tries to formulate and to apply the method of informatics (top down) to the world of business.

Teachers' Training in the Federal Repulbic of Germany consists of three stages:

(a) Education at a university leading to a university degree and state certification
(b) Subject certification
(c) Subject certification for teachers not especially trained in the field of Business Informatics

Vocational Training

In the Federal Republic of Germany (FRG) the education of students in vocational schools is a double track education. It takes place simultaneously in the vocational school and in the actual world of business. Although the general guide lines for this double track education are given by the federal government centrally, the final authority and implemntation lie with the federal states.

The synchronisaton of these two authorities is very complicated and time-consumimng. Normally the implementation of a new curriculum takes 6-8 years.

During the course of his studies at a vocational School a German student is taught at total to 30 - 40 hours of Business Informatics.

Problems:

- Great flexibility is required in vocational schools in Germany, because students come to these schools with very different educational backgrounds (high-school certification, Senior Matriculation).
- At the present there does not exist a formulated systematic didactics of Business Informatics for future vocational school teachers. The limited number of hours (30-40) allotted to the subject of Business Informatics at vocational schools does not adequately prepare the student for his future practical work in the world of business.
- Since no didactics of Business Informatic for future vocational school teachers does exist, neither lectures nor seminars are offered in the subject.
- The Cooperation between unversities, vocational schools and the world of business is practically non-existent.

Teachers' Training in Universities

At present only two universities train future vocational school teachers of Business Informatics. However, a few universities are planning to establish such programs.

It is highly desirable that all future vocational school teachers - regardless of their subject specialisation - receive extensive training in the field of Business Informatics, up to the level of certification

Problems
It takes at least 8 - 10 years for such innovative programs to become effective in vocational schools.
At the moment the majority of future vocaitonal school teachers do not receive any training in Business Informatics at the university

Advanced Training of Teachers

Considering the urgency to find quickly vocational teachers for the subject of Business Informatics a crash program was highly desirable.

Momemtarily, vocational school teachers already certified are given the opportunity to acquire the certification in Business Informatics by participating in a crash program. This program consists of four courses of one week each in length.

Just as in the basic training there exists hardly any cooperation between the university, vocational schools and in the world of business as far as a advanced training is concerned. The crash program puts heavy pressure "financial, organisational, personal" on the institutions involved. The crash program suffers from a lack of contacts with the world of business. It is too theoretical.

Teachers' Training in "Studienseminaren"

In Germany the supervised training for teacher candidates in the classroom only takes place after graduation over a period of 18 months. This is called a "Studienseminar".

There exist far too few "Studienseminare" for Business Informatics teachers. The lack of an existing didactics for Business Informatics makes the work in the "Studienseminar" exceedingly difficult. It amounts to a trial and error effort.

Additional problems:

- In order to acquire the desired "computer literacy", manuals, text books, computer programs for teachers as well as students etc. have yet to be written or developed.
- The exisitng shortage of computer installations in vocational schools has still to be overcome.
- There exists a great discrepancy between the capacity of computers used in business and those installed in vocational schools and used for training.

5.2.3 Teacher Education on Computers in an Open System

The group noted that we are training teachers who will teach for 30 or 40 years. We cannot give them a definitive body of knowledge, but we have to enable them to adapt and evolve. At the present state of development there is a strong link between pre-service and in-service training. In-service training is in fact "pre-service" because of the constant evolution of informatics and pedagogy. Computer science is more than a technical tool. It is also a tool for thinking, reasoning, and problem solving; it can lead to different thinking styles; questons of means are as important as ends.

Computer science is embedded in the curriucla, it is not a complement to it. Student teachers must be taught: Not only computer science, but also pedagogical aspects of its use.

Not only programming but also algorithmic thinking.
To re-centre in mathematics the role of algorithms.
To stress the main algorithmic concepts and methods of problem solving.

The different ways of informatics appear: data banks, word processors, expert systems etc.

5.3.2.1 USA: Didactical Studies of Mathematics Teachers

L L Hatfield

Based upon the rationales for, and characterizations of, mathematics instrucitonal computing described above, several guidelines and qualities of the didactical preparations of mathematics teachers can be posited.

(a) To develop a beginning didactical knowledge for instructional computng in school mathematics, the teacher must experience computing as a student. It is essential that we recognize that today's teachers have not themselves learned mathematics using the computer as an integral tool. The teacher must be allowed to study mathematics in contexts wherein the constructions of his/her knowledge involve various types of comnputing. This suggests that we must begin the integration of instructional computing into the mathematics courses of the teachers' programs of studies. And within professional courses, such as the study of curriculum or pedagogy, the teacher must experience treatments of school mathematics instruction in which the teacher educator models prototypics applications. By exploring concepts and generalizations, by solving significant problems, and by practicing skills which they themselves are engaged in constructing, teachers should experience and come to understand the potential influence which effective instructional computing may have upon a learner.

(b) Specific attention must be given to developing didactical knowledge for mathematics instructional computing wihtin extant professionalized education courses. What is sought here is the general knowledge of all mathematics teachers for conceptualizing, planning and conducting lessons in which they integrate various types of computing aplications. Within contexts of studying curriculum and teaching, teachers would explicitly examine the purposes and strategies for using the computer in instruction. In particular, the five areas of issues discussed in the previous section, "Developing Effective Instructional Computing". must be carefully addressed. For example, in a " methods of teaching" class which deals with the teaching of problem solving, the teacher might participate in a "demonstration" lesson in which the group writes, tests, refines, and generalizes a computer program for iteratively approximating solutions to the equation, $x + 4x + 1 = 0$.

The ensuing discussion may focus upon the curricular implications of such "solution" processes in school mathematics, or upon the pedagogical tactics for guiding students to develop together a computer program.

(c) Mathematics instructional computing specialists must be prepared through post-graduate teacher education studies. Beyond the general, background didactical knowledge for instructional computing which all mathematics teachers shoud develop, there is a growing need for specialized leadership expertise in all sectors of mathematics education. In many countries there is a trend toward post-graduate certificates or degrees featuring a generalist treatment of educational computing, perhaps reflecting the current demand of a person to guide computer implementations in all subject areas of the school. However, the complexities and urgencies of integrating computing into school mathematics are now sufficiently critical that we must accelerate the leadership effect by preparing instructional computing specialists specific to school mathematics. These persons would become "resource teachers" to assist inservice teachers in becoming knowledgeable for using computers in their classroom. Their preparations would include mathematical and computer science foundations, such as understandings of discrete mathematics, iteration and recursion, difference equations, and the "algorithmic approach". More importantly, they would be deeply prepared through advanced didactics studies to integrate student-programming approaches into both traditional and new topics in school mathematics. Their teaching methodologies would emphasize a problematic approach in which the construction and refinement of an algorithm (i.e. a computer program) is a significant aspect of understanding mathematics.

These specialist "resource teachers" must also be knowledgeable of various forms of computer-based instruction, including the psychological and pedagogical bases for designing and using such resources in teaching. Their knowledge of available educational software should guide the critical selections and thoughtful implementations needed in a mathematics curriculum in transition. They should be able to assist in adapting and installing software which enhances the numerous roles of the classroom teacher. With a solid background in computer programming, instructional design, and mathematics education, these persons should be able to create and improve effective instrucional software for themselves and other mathematics teachers. Furthermore, they should be knowledgeable of sources and uses of various teacher utilities, and be able to assist teachers in selecting and installing such electronic aids for designing or administering tests, maintaning a gradebook, or preparing printed communiques.

5.2.3.2 France

B Cornu

The question of computers and informatics in the pre-service training is very crucially posed in France. One of the main problems is to know what must be taught to the future teacher or in the future to the teacher. Nobody knows what informatics will be like 30 years from now; we are training today teachers who will teach during the 30 or 40 years. It is impossible to give them a body of knowledge for their whole career in advance. So the main aim of the training is to give them some foundations, so that they are able to evolve and adapt, all through their career. This is the reason, why it seems to me that it is impossible to solve the problems of pre-service training of teachers

Some points about the pre-service training:

First of all, of course, the teachers have to learn a little informatics! But many questions are posed: How far must they go about the hardware and the structure of computers? Do they have to learn only one language, or several? Which ones? Is it necessary to have previous training in algorithmics, so that they are able to distinguish between algorithmic problems and language problems?

At the present time, we do not know what teachers will have to teach about informatics; neither do we know to what extent they will have to write their own programs by themselves. "Expert systems" for teaching begin to appear; they enable the teacher to build pedagogical uses of computer to his liking without programming (just like how the word processors or file processors ...?). Then it will be no longer necessary to know programming in order to be able to use computers for teaching.

Second aspect: training for pedagogical uses of computers. The relationship between the computer, the pupil, the teacher may be dealt with in several different ways. What can the teachers' part be? What part of activity and autonomy does the pupil have? Do the pupils write their own programs, or do they use existing programs? Between these two extremes, there are a lot of possible pedagogical behaviors, and one has to improve technics so that the pupil may be active. Some open-ended software begins to exist.

Third aspect: The problem of the influence of informatics and computers on mathematics themselves. It is sure that informatics introduces changes in mathematics and in the evolution of mathematics. Computers and informatics have stimulated new research, have called to the mathematician's attention questions recently neglected but previously studied over a long period of time, and have made possilbe the study of new questions. It also make some topics obsolete. These points are strongly stressed in the first report of International Committee on Mathematical Instruction (ICMI) on the influence of computers and informatics on mathematics and its teaching

But informatics also changes the mathematics to be taught. For example, what must be the place of discrete mathematics in teaching? Do we have to introduce calculus through discrete mathematics and the concept of approximation? Another example: Symbolic systems and automatic fomal computation are quickly developing, and will be available for everybody in a few years. What shall we have then to teach about integration, derivation, polynomical computation? Questions will be raised, similar to the one which was often asked when pocket calculators appeared: "Will the pupils be able to compute by themselves?"

Finally, an essential point is the status of experimenting in mathematics. Computers give us an efficient tool for manuipulating and experimenting. Before introducing a new notion one can carry out experiments, so that the pupils get a previous intuition and are able to make some conjectures, to make many observations, to visualize complex mathematical objects, to simulate situations. The observations may lead to elaborate a formula, a theorem, that one then can prove formally. With the computer, the pupils will have the reflex of trying, experimenting, and of numerical verification.

5.2.3.3 New Zealand

Ch. Greenbank and B Doyle

The future in New Zealand for Computer Education will see the advance of the present situation without radical change. Government has recently give a commitment to fund hardware purchases in Secondary Teachers Colleges and Technical institutions. Plans are being made to equip senior schools with hardware through subsidy schemes. No plans or policy exist for primary schools or for primary teacher education. A syllabus of Computer Awareness will be introduced soon for all children at lower secondary level and in

Computer Studies for selected students at upper secondary level. In-services and pre-service training for those teachers working in computing studies will develop and the vigorous informal activity which at present characterizes the New Zealand scene will gradually become formalised into the national education system.

5.2.3.4 Germany
Consequences for Teacher Education
K D Graf

The different models mention in 5.2.2.3 ask for different ways (as to extent and contents) of pre-service teacher education.

(a) In the case of integration there is consensus that a regular subject of studies "basics of informatics and its applications" has to be included into the curriculum of prospective math teachers and/or physics teachers etc.

The following requirements seem to be reasonable:

- knowledge and skills in algorithmics, the emphasis being on their importance as a mathematical method. There should be awareness of the value of the algorithmic method,
- for the development of mathematical concepts and methods and for effective problem's solution,
- for a cognitively efficient representation of problems and the respective solution methods,
- for the treatment of numerical problems;
- knowledge and skills with regard to at least one higher programming language, and to the methods of sturctured programming. In addition familiarity with the basic structure of computer systems.
- Knowledge and skills concerning applications of data processing relevant for practival purposes. This is important, above all, for fields removed from mathematics and the sciences. This includes an overall view of the social relevance of data processing ,computers and informatics.

A curriculum fulfilling these requirements has been developed for the teacher education colleges in Baden-Württemberg and is being practised now. It has to be combined with studying mathematics as a main subject. About 3 to 4 lessons a week have to be studied for six semesters.

(b) In Berlin, studies in data processing can be integrated into mathematics teacher training, one lesson a week for six semesters on an average. Ay present a program for "full informatics teachers" is being introduced at the Freie Unverstiät Berlin, demanding approxmately 10 lessons for six semestes. (Teachers have to study two school subjects, each with 10 - 13 lessons for six to eight semesters, plus educational sciences). The studies include compulsory areas like

- programming methods, including foundations of software engineering
- elements of computer organization
- applications of informatics and data processing, including their social consequences, especially in education.

More, the students have to choose two out of five elective areas, namely:

- data processing user systems in business, administration and law
- data processing user systems in technics and science
  software technology
- computer organization
- theory of computer science/informatics.

It is expected that students graduating after such a program will become teachers, who will be able to teach informatics not only as additional subject matter. Their teaching should bring about

- ways of thinking which can be applied between different disciplines
- de-mystification of information technology by discovering its tool-like character
- discussion of the social consequences of the introduction of information technology.

5.2.4 The Effects on Mathematical Education Subject Matter, Methods and Ways of Learning and Teaching

Computer Science changes the philosophichal foundations of mathematics. For example:

Impact on what mathematics is to be taught.
The development of symbolic systems.
More experimental mathematics.
New skills - obsrervation, visualisation, simulation, conjecture and numerical verification etc.

Computer science changes pedagogical styles. The computer must not be used in ony one way; we have to carry out a multi-useage appraach. Nor must the computer be used to do things that can be better done without one. The computer does not replace the teacher but will alter the pupil-teacher relationship. The teacher must become a partner of his pupils.

Classically:	TEACHER	Now:	TEACHER	COMPUTER
	PUPIL KNOWLEDGE		PUPIL	KNOWLEDGE

5.2.4.1 USA

L L Hatfield

Background

It is now necessary to undertake a thoughtful, but accelerated evolution of what purposes and meaning for what we refer to as "educating" for mathematics in the schools. This assertion is based upon a variety of forces for change converging upon the activities we currently know as the teaching and learning of "mathematics".

1. The ubiquitous microprocessor. Many domains of human acitivity are already deeply engaged in the implementation of computers as tools. These events may be characterized as transformations, which in serveral domains are virtually revolutions. The scope of these transformations is only vaguely understood, even by those who are participants. The effects of these impelmentations of computers throughout a society are speculative at this point, though some believe that "the greatest significance of the computer lies in its impact on Man's view of himself" (Simon, 1977). The "symbiotic evolution" of man and the computer discussed by Kemeny (1972) appears now to be our destiny.
2 Scholary research. The subject matter disciplines (which consitiute the bases of school curricula) are being influenced by the applications of computers as tools. Scholars in essentially all disciplines are using computers, often in many different ways. These usages not only support the conduct of traditional research and development, but the nature of a computer-oriented enviornment often influences the questions posed, the investigative methods employed, and the qualities of the evidence of ideas generated.
3. Mathematicians and computers. Mathematics is a complexity of human activities serving many varied purposes. In a strict sense mathematics is founded upon numerical and spatial reasoning, yet the paradigm of mathematical maturity is often purely symbolic formulations unconstrainted by specific "real" numerical or spatial referents. Early applications of computing emphasized increasingly sophisticated numerical procedures and methods, and the evoluton of computers has paralleled the emeregence of many new domains of mathematical research, encompassing both theoretical and applied emphases. At the same time, recent computer software developments employing powerful symbolic algebraic, trigonometric, and calculus procedures now provide interactive, computer-assisted manupulations of mathematical formulations traditionally constructed only by humans.
4. Views of knowledge. What it means "to know" and "to do" mathematics is influenced by the general orientations operating within society, as well as by the disciplined activities of the research mathematician. The epistemological perspectives greatly effect the conduct of mathematical education: its goals, ideas to be taught and learned, and methods of teaching and learning. The capabilities and potentials of the computer present new confrontations with our traditional expectations regarding the knowing and doing of mathematics by teachers and students. Some of these challenges relate to different ways of approaching traditional content, but others involve our consideration of new mathematical contents in the curriculum.
5. Mathematics teaching. The roles of the classroom teacher in effecting quality learning continue to be matters of debate and study. The dominant pedagogical constructs for describing or explaining classroom teaching and learning have placed the teacher in the role of informaed discussant, disseminating through presentations and textbook-centred tasks the critical information with the student receives, stores, and processes. In this sense, "mathematics" has been something to be transmitted through the expressions of a knowledgeable teacher, supported by

resources such as textbooks and chalkboards. With computers allowing the development of increasingly complex "informational systems" to be accessed by persons using mathematics, the traditional instructional modes of information processing are now becoming challenged.

6. Mathematics learning. The roles and strategies which mathematics students have adopted in order to learn mathematics have been largely reactive and imitative. In the view-point of many students, much of the content emphasis is upon the "how to", and when mathematics teachers try to encourage thinking which seeks to make sense of the generalizations and procedures, students often respond with a deep awareness that the tested performance will only require demonstrating correct answers form standard algorithms. The students' behaviors for learning often reflect a conception of mathematics as body of information to be received through the presentations of a teacher or a textbook, to be practised through examples and exercises patterned after those presented, and to be remembered at least until the next test is completed. Few mathematics students seem to view the learning of mathematics as a context for developing their intellective abilities or generalized problem-solving

competencies, as they become mired down in the daily, piece-meal rituals of practising and remembering. It is this conception of mathematics (partly culturally transmitted but primarity due to their actual classroom experiences) which effecitve instructional computing might hopefully alter.

The assumptions express the opinions that computing in society and in scholarly research will be increasingly omnipresent, that our epistemological views of mathematics must be altered because of computing, and that the contemporary expectations and approaches to teaching and learning mathematics must now be critiqued and revised. Three inter-related frameworks for developing instructional computing are summarized below.

Developing Effective Instrucitonal Computing

What qualities of learning and teaching should be sought in our attempts to develop mathematics instructional computng? We face many issues and difficulties, among which the following are highlighted:

1. Computing as an "end vs. means". The impact of the microcomputer throughout society has sometimes produced a near hysteria among educators In many schools the machine itself, and the study of it, has become the dominant focus, as "computer literacy", "computer math" and "computer science" courses are hastily added. Setting computing apart from the essential mathematics curriculum may provide students with experiences which result in them never learning to use the computer as a tool for explroing and constructing mathematical ideas. Rather, isntructional computing as a "way of life" in the mathematics classroom would provide integreated experiences in using the computer as a means for learning. On the one hand we may have a student who rarely cosiders using the computer unless explicilty told to do so, for computing is something done in a "computing course". Or we may have a student who routinely turns to the computer when faced with all manner of problematic or mundane mathematical tasks. Computing must beocme integrated into the mathematical life of the student, early and continously throughout schooling.
2. Eclectic enrichment vs. curricular embeddedness. Our explorations of instructional computing must be pursued with a clear committment to building curricula wherein the computer is assumed as an integral tool. Day-to-day constructions of knowledge are approached through various usages of the computer in the classroom. For planning lessons the teacher would routinely ask: In what ways might I use the computer to help my students understand, and become operational with, the mathematics of the day? For exploring algorithms the approach may involve student programming tasks. For examining a variety of instances leading to the formulation of a generlization one might use a program for demonstrating. For consolidating newly formed concepts or practising certain skill-oriented actions a gaming usage may be called upon. For modelling an application of the mathematical ideas in a "real world" context a simulating program would be used. Computer-based instruction programs for testing, informing, tutoring or communicating usages might be

incorporated into the lesson. Whatever instructional computing usages are chosen, they offer students experiences which are anchored in the goals and contents of the essential mathematics curriculum.

3. Modelling/imitating vs.guiding/constructing. With an actual impact of computers into the classroom the role of hte mathematics teacher must change. These changes will surely involve a gradual shilf away from the teachar-dominated pedagogy where the teacher and the students expect the teacher "to know all, tell all, show all, and do all", with the expectations for the student to listen, to watch, to imitate and to remember. Teachers, if indeed the profession is to survive, must shift toward the role of mentor, serving to stimulate and guide each student in the personal constructions of mathematical knowledge. More routine instructional supports will be found in the student calling upon the computer in various ways for information, for demonstrations, for practise exercises, for word-processing, for limited tutorials, or for simulations. The interactions with the teacher, individually and in peer groups will be characterized by "getting started" on major blocks or problem areas of mathematical study which will be apprached more as a project by the student. To do this, teachers will have to be ready to pose significant, overarching questions and to guide students in their searches and refinements of these questions. "Problem posing" will become much more important to the actions of the teacher with the phases of problems refinement and problem solving becoming much more the province of the student. More attention to heuristical emphases in the teacher/student interations will also be required.
4. Singular vs. multi-usage approaches. With little didactical knowledge for instructional computing teachers still often adopt a single type of computer application for their instruction. Perhaps the most dominant use is practising, this often being rather peripheral to the mainstream of classroom activities. Instructional computing as we may now envision it can incorporate a great variety of distinct types of usages. We must explore and captilazie upon all the possible ways the computer might assist the teacher and the student. Thus, as we seek to build computing as a means to teach and learn, as we strive to anchor our computer usages in the essential mathematics curriculum, as we adopt constructive mehtods and problematical approaches, we must be alert to a multi-usage perpsective. Our vision might be a classroom in which the teacher and the students are using various computers tailored to the demands of the task. These usages span many types of instructional applications, with different students pursuing different activities during the same time period. The computer is viewed as a multi-dimensional tool, and to accomplish the goals of instruction we try to fit the particular tool to the task.
5. Displace/replace vs. central/essential. We must recognize the absolute essentiality of the knowledgeable, sensitve mathematics teacaher for effective instructional computing. These marvelous machines cannot replace the effective human. Rather, our "symbolic evoultion" must be toward sharing roles in the instruction of mathematics. Frankly, it may be reasonable to allow computers to perform certain teaching tasks (such as guided practising) which have been prominent in the actions of

mathematics teachers in the past. This will free the teacher to feature situations involving higher order thinking, to interact more personally with each student, and to probve for more penetrating understandings of what each student knows, wants to know, and may come to know. This will require us to develop more clinically oriented skills, and to origanize our schools to allow for such individualized interactions.

5.2.4.2 An Informatics Course for the Teaching of Mathematics

B Cornu. France

Last year, we have introduced a course at GRENOBLE, whose title is "Elements of Informatics for the Teaching of Mathematics". This course takes place in the "matrise", that is for students in their 4th year at University, who are future mathematics teachers. Inside the "matrise", there exists an option (say "Preparation for teaching") with two courses:

- Didactics and history of mathematics
- Informatics for the teaching of mathematics

It is an optional course. It consists of 3 parts:

(a) Initiation to informatics and computers: elements of algorithmics, studly of the PASCAL language; a little BASIC language. This part is very classical. The content may vary according to the initial knowledge of the students (more and more, they will have studied this previously, but it is not the case at the moment).

(b) Numerical mathematics. In this part, we study several examples, showing how a mathematical problem can be approached through an algorithmic and numerical way. The aim is to change the students' outlook about mathematics, which is often very formal and abstract. We want them to get an experience of observation, experimentation, conjecture, visualisation, simulation with the use of computers. The examples we present are rather classical: Convergence acceleration for sequences and series; iterative methods for equations and systems; difference equations; differential equations (numerical and graphical methods); integration; algorithms for matrix computation; polynomials and interpolation; fast Fourier transform; simulations (specially in the field of probabilitites and statistics). An initiation to symbolic systems is also included.

(c) Informatics and teaching. This is the really new part of the course. It includes:

- examples of using informatics in the teaching of mathematics;
- pedagogical and didactical aspects
- study of existing software
- study of the LOGO language and its use in primary schools
- how to conceive teaching-software, and the pedagogical and informatic problems pertaining to them
- expert-systems for teaching
- visits in classes where computers are used
- and then, each student has to work out a personal project: a didactical approach of a mathematical problem, and implementation on a computer (writing a program; technical and pedagogical documents accompanying the program).

At the end of the first year of this course, the students have expressed the wish that the 3rd part be amplified. Next year, we shall stress more the use of LOGO, and we shall try to develop some scenarios for teaching software together with the students. The latter will have to attend secondary school classes using computers, and to participate in the elaboration of this use. It is also foreseen to give the students elements for evaluation of teaching-software. Finally the psychological and cognitive aspects of the use of computers and informatics will be of further study.

Next year, the course will be given simultaneously in other subjects: biology, geography, physics, and we shall have several common lessons. So the approach will be interdisciplinary, and we hope this will be very interesting at the pedagogical level.

5.2.4.3 A New Subject "Computer Education" or "Computers in Mathematical Education"?. Germany

K D Graf

There is still a strong controversy whether informatics should be taught as an independent subject in schools or whether teaching how to use computers and other methods of informatics will be more effective. In higher secondary education the first way is dominating. In lower secondary education different models are exercised in different state of Germany, due to the federal stucture of the Federal Republic of Germany, e.g. in Berlin (West) some schools offer "informatics " as a "compulsory elective subject" in grades 9 and 10, about 220 lessons. In Bavaria in many " Gymnasium" courses in programming of about 35 lessons are integrated into mathematical education in grade 10. In Baden-Württemberg an interesting model for the integration of basic facts about computers and informatics into differnt other subjects has been worked out. Under the heading of "technology", topics are dealt with like 'application of computers to the solution of practical problems', 'construction and use of simple electornic devices', 'construction of a computer controlled device and design of a program'. "Mathematics" includes 'work with a computer on applied calculus'. "Social Science" deals with 'division of labour and automation' in grade 8. Similar aspects are dealt with at qualified secondary schools in mathematics, physics, natural sciences, technology and social science, carrying as far as 'chances and limits of modern information technology'.

5.2.5 Problems in School and Society

Several problems exist:

(a) How to convince our colleagues of the role computer science plays in mathematics.
(b) The need for serious reflection upon the relationship between mathematics, computers and education.
(c) How to disseminate our findings knowing that we're already late with respect to the social dissemination of computers themselves.

The working group expects considerable turbulences in the development of mathematical and computer science education due to the rise of information technology. Many problems cannot even be foreseen.

National and worldwide meetings will be helpful in identifying true directions. We must also identify blind alleys, fads, and those directions wasteful of our time, energy and resources and destructive of students learning and mathematics.

The working group is also aware of deep philosophical questions which information technology raisies in our society. Teachers have to be alert and ensure that students learn how to use information technology as a powerful tool and how to contol it completely.

5.2.5.1 How Do We Live With a Computer and Love it Without a Heart?

Republic of China

M Tong

Allow me to put the question in a different and in an ignorant sense of direction: what should we ask, is the nature of human beings that God had in mind when creating ADAM? And what then is the nature of the computer? And does it run out of hand after the Creation and the Garden of Eden?

Indeed. What is a computer? How do common people like me look at it without understanding the black box inside? What relationship does one try to establish between oneself and a computer? It seems that no such 'monster' existed in human history before. An automobile may serve for a similar argument. But soon one seems to conquer it and evetually take it as either an extensin of body or else as a good companion. But as to a computer it is hard to treat it as part of us. As a matter of fact one respects it, utilizes it or even plays with it. Yet hardly one would love it. That is not the thing to spend sentiment on. We know we have to live with it, it will stay.

Computers play the role of answering man in our life. That is true, unfortunately, it started with novels like '1984' and movies like "Star Wars". A computer is a god of no grace, a human unkind par excellence, a dark mask of technology. Computers after all will soon replace humans and take them as slave-turning regrets. We all know that once invented, it will never vanish from human nightmare again.

Now our problem is, if 70% of the above observations do appear to be right, how do we, deep in our heart, come to teach our little ones to learn and to live with a computer?

Let's face the fact that most of our teachers in today's computer teaching professions have nothing to do with the computer industry. They teach because they have to teach, and thus the educators have to offer a so-called pre-service training to them. They should evenutally learn some skills of operating. Yet hey are in a position worse than their colleagues in the NEW MATH situation in the sixties. They teach what they have just learnt from someone of the same category. They do not know what is behind and what is in front of them. Let me just pose some questions here and hope for positive answers in the future.

a) Do we just teach children to use computers as to replace human hardship of old time in Mathematics?
(b) How to estimate the limitation of human creativity that is caused by computers and computer teaching.

(c) The society may soon depend on the operation of computers. How then do we face the fact of domination and the aftermath of computer pollution?

(d) As Society marches forward, how do we consider the problem of instrumental season (in the sense of Horkheimer, Adorno, Marcuse and Frankfurt School), that computers serve as a perfect example for the technical domination of nature over man, which alienates human life, human needs, as well as human intellect?

5.2.5.2 **USA**

L L Hatfield

The advent of the computer in the classroom is causing us to stuggle with fundamental questions of what is knowing and doing mathematics, what is educating for mathematics and for mathematics teaching, and what are the roles of the mathematics teacher and teacher educator. To avoid such basic questions could result in a technological coup d'etat in which traditional teaching fades away as an instructional dinosaur of the pre-computing era. In dealing with the challenges of developing effective instructional computing we mathematics teacher educators must recognize how essential it is that we transform our own teaching. To build toward a rich, humanistic utilization of the computer in mathematics education will require thoughtful, visionary planning. The choice is not between the computer OR the teacher. The only acceptable, and more promising, paradigm is the computer AND the teacher whose didactical knowledge permits effective usages of the machine to enhance learning and teaching mathematics.

That school mathematics may easily become the major battle ground between technocrats (whose vision of schooling involves the administration of computer-based courses) and educators (who deeply believe in the centralitiy and essentiality of the human, adult mentor/teacher in nuturing subject matter knowledge as an individual, yet social, construction) is apparent in the arguments of at least one leading computer advocate. Bork contends that computer-based courses in mathematics should be created because these "... can compete very well .... with what typically happens in our classes at all levels", and because of the shortage of qualified teachers"... the need is great to develop self-standing computer based courses, which can be used without teachers".

6 MISCELLANEOUS

6.1 How to Start with a Computer

6.1.1 Introduction

6.1.2 Hardware Facilities
6.1.2.1 Working with Computers
6.1.2.2 Minicomputers in Schools
6.1.2.3 The Microcomputer

6.1.3 Didactical Aspects of Beginning with Computers in Mathematics
6.1.3.1 Introduction
6.1.3.2 General
6.1.3.3 Programming

6.1.4 Organisational Aspects
6.1.4.1 Introduction
6.1.4.2 Location
6.1.4.3 Scheduling
6.1.4.4 Timetables
6.1.4.5 Aids for Children
6.1.4.6 Tasks

6.1.5 Software to Serve Students
6.1.5.1 Advantages of Software Standardisation
6.1.5.2 The Need for Diversity
6.1.5.3 Differences among Students
6.1.5.4 Different Types of Software
6.1.5.5 Matching Software and Students
6.1.5.6 A Recipe Book Approach
6.1.5.7 Learner Controlled Software
6.1.5.8 Author Directed Software
6.1.5.9 Summary
6.1.5.10 References

6.2 Technology for Education of the Handicapped

6.3 Technology as an Aid to the Teaching of Problem Solving

6.1 How to Start With Computers

Gustav Vonk and Michael Wheal

6.1.1 Introduction

The question about the exact meaning of this title rises before long. Does it suggest the demand for a checklist to be used by schools when purchasing computers for education? Or suggest a survey of the various possibilities that exist for providing computer facilities in education? The latter interpretation will be dealt with in sub chapter 6.1.2.

Yet another interpretation of the title is: How can computers best be introduced to pupils? Applied to this conference we would add to that "in the mathematics lessons". It is clear that no generally valid answer is possible. Only a number of experiences of the writers and other members of the working group can be collected.

Each and every contribution expresses the complexity of the material. So that the reader will not be lost in the multitude of aspects, a rough division has been made according to the three facets that were most evident during the sessions of the working group:

6.1.3. Didactical aspects
6.1.4. Organizational aspects (case study)
6.1.5. Software to serve students

Since this division is not exhaustive, some noteworthy remarks will be included in the text which are not covered by the pertaining sub chapter title. On the other hand the working group did not represent world-wide expertise so that the text will often relate individual experiences that cannot as such be transfered to other countries and other education systems. But no matter how specific and dependent on time these experiences may be, they are exemplary for that which many will experience as : "How to start with computers".

6.1.2. Hardware Facilities

Developments in automation are of course reflected in education. One sometimes gets the impression that education circles accept the novelties in the field of computers more readily than the business world does. At least as far as the rigorousness with which existing possibilitites in education are thrown overboard for the sake of newer trends. Something which is foreign to society as a whole. These trends can also be observed in consecutive ICME's.

As far as computers were mentioned at all at the first ICME, held in Lyon, France (1969), the trend was towards the use of computer centers - sometimes located at universities, sometimes specifically for primary and secondary education. (1)

The conference at Exeter revealed two new developments: "desk machines which require the use of a machine language and mini-computers or terminals on which algorithmic languages can be compiled" (2). It was also at Exeter where Seymor Papert demonstrated his 'turtle geometry' on a mini-computer with display terminals.

By the time Karlsruhe (1976) came along, the interest for desk machines had disappeared in favour of the programmable calculator (3). The strongest statement that was made in this area at Berkeley (1980) was uttered by Arthur Luhrmann: "terminals are out, micro's are in" (4).

Two trends can be distinguished here that are not parallel but which will intersect one day.

The one is, as mentioned before, induced by the de-centralizing developments in automation. The computer facilities have transferred from central (national or regional) to mini computers in the school. Both offering the possibility of the use of card input and/or of terminals. This was followed by the micro computer in the classroom. The next step is the briefcase or pocket computer for the individual pupil.

From the outset the second trend involved the distribution of cheaper equipment to the pupils. Education put the scarce possibilities of such equipment to the best possible use and with the increasing capacity of such 'hand-held' devices, the educational applications were also improved.

Adelaide did not yet indicate the convergence of these two trends. Perhaps we will have to wait until ICME 6 or 7 to be confronted with the computer for the pupil which has the dimensions of a textbook and the capacity of the mini computer of today. Only the shared facilities such as printers, plotters and coloured display screens, will be permanent in the school, accessible from each school desk via a local network.

Large data banks are once again set up regionally and can be reached via the school network. The pupil's computer serves as a terminal from which education can be received in the event of illness.

The reason education is so quick to discard previous technical solutions is usually due to the disappointing applicability in education. This disappointment often in the area of organization, insufficient didactical in sight in the kind of computer facility as such. It is intentional that the foregoing paragraph portreys a future that will make use of precisely the strong aspects of computer centers, school computers, classroom facilities and computers for pupils.

6.1.2. Working with Computer Centers

The use of existing computer centers does of course lie within the decision authority of the school. The appointment of new centers or the adaption of existing centers, specially for education, is a task of a higher order. Computer centers not specifically designed for education - university computer centers usually belong to that category as well - have the disadvantage of not being able to offer the desired to the pupil. One must adapt to the current system and the software which is available often displays barriers which the pupil cannot take. In computer centers which are intended for secondary education, these thresholds can in most instances, be removed.

All computer centers are faced with the problem of initiating the communication between the pupil and the computer. Two primary possibilities have emerged until the present: terminals or mark sense card input and printed output.

Disadvantages:
- high telephone costs usually restrict the number of terminals per school to an unacceptable minimum.
- no graphics or colour screens on the terminals.
- the turn around time of the cards restricts the educational possibilities, in computer science to a lesser degree, in mathematics and such to a large degree.
- the error percentage is high for the cards.

However, computer centers for education at a regional or national level have advantages that sometimes tend to be overlooked because of the disadvantages.

+ the center collects all kinds of software and exchange is no problem.
+ the center is capable of setting up data banks for educational purposes while smaller organizations cannot.
+ still the least expensive solution for many applications, under the condition that there is sufficient work load and that computer time can be sold to third parties during vacation periods.

The Educational Computer Centre in The Netherlands has proved that it is possible to operate on the basis of US$ 0.40 per pupil activity on the computer. This is for a mini computer with 100 000 pupil actions per year, occupied to one quarter of its capacity.

The same computer center succeeded in reducing the earlier mentioned disadvantages of communication considerably.

= Terminal and cards & paper can be used in every combination. For example, input by mark sense cards in bulk and corrections and output via the terminal or input by terminal and printed results by mail. In this manner a limited number of terminals can be put to optimal use.
= Errors on mark cards can be reduced by half by providing the possibility of direct correction on the card, by error recovering software for reading the card and especially by abandoning the Hollerith codation.
= Software directed at a minimum of input without becoming cryptical. For the programming languages used by the

pupils this means the standard presence of many strong procedures so that pupils can suffice with few procedure calls instead of ten-fold text in corresponding Basic programs for example.

6.1.2.2 Mini Computers in Schools

The advantage of mini computers in schools is the employment of these machines for organizational and administrative purpose of the school. However this computer will not have the data-base possibilities of a larger computer center. The excahnge of software will often be impossible. Mark sense cards and terminals will often be found to be the means of communication between the pupil and the mini computer. The lack of telephone costs makes exploitation of the terminals less difficult.

Improvements of the system in the spirit of tne previous sub chapter can greatly increase the efficiency here as well.

6.1.2.3 The Micro Computer

As quoted in the previous, "micro computers are in, everything else seems to be out". This rather abruptly passes by the advantages of older generation computer facilities. It is not impossible to provide a card reader for a micro computer and to introduce the previously described solution of central processing in the classroom. Do not overlook the considerable advantage: all of the pupils in the class can be occupied simultaneously in preparing the computer input, thus fairly well eliminating the bottle neck of the keyboard. In The Netherlands there is experience in this respect with an operating system for a micro computer whereby pupils run their cards through the reader, while a continuous batch process is maintained of everything that is introduced by the pupils, without human intervention. This requires no reorganization of classrooms nor of classical instruction.

This system obviously offers less interaction between pupils and the computer. In mathematics education the application possibilities are restricted to standard programs without any interaction (e.g. programs for matrix multiplication, simplex method), the study of numerical methods in which computer programming is merely a conformation of earlier studies and for simple programming to show that students understand certain concepts. For other applications it is precisely the interaction that is deemed important. This brings us to the subject of didactics and therefore the next sub chapter (6.1.3).

In the rest of this chapter we will focus on the most commonly used computers: the personal microcomputer.

Any discussion of beginning with these computers will ultimately consider funds, hardware and software.

Initial costs will continue to be a problem but perhaps they have always been minor in comparison to ongoing costs of maintaining, operating, updating and supporting the system.

From the viewpoint of mathematics alone the hardware requirements are simple and perhaps even a minor consideration: robust construction, ease of use, adequate memory (48K), disk storage and simple graphics.

The potential to add other peripheral devices and use them conveniently should be considered but perhaps the most important factor to note is the quality and availablility of software for the hardware configuration chosen. See the last sub chapter.

(1) Proceedings ICME I, page 223.

(2) Proceedings ICME II, page 57.

(3) A Engel, "The Role of Algorthms and Computers in Teaching Mathematics at School", Proceedings ICME III, pages 83 - 88.

6.1.3. Didactical Aspects of Beginning with Computers in Mathematics

Michael Wheal

6.1.3.1 Introduction

Innovations in teaching usually occur when there is a desire of opportunity to improve the quality of student learning. The introduction of computers into schools is usually justified or rationalized on such grounds. In the past many schools and teachers began by obtaining some hardware and then sought to discover, through experiment, what could be achieved. This was not only understandable but quite reasonable given the lack of specific knowledge about the ability of computers to improve the learning of real students under the supervision of real teachers. There may not yet be any experts but there are now many teachers who have made enough mistakes to have learned from them. No doubt teachers, schools and school systems will continue to buy beofre seriously considering the effects upon their curricula. There is a cost involved in any change. With computers the cost in terms of money, time, personal energy and creativity, to name but a few items, can be considerable. Because of its general power and nature, the computer brings almost unimaginable potential for change and teachers have no obligation to ensure that these changes are of benefit to their students.

6.1.3.2 General

Before introducing computers (or computing power) into classrooms, those responsible for the curriculum should be quite certain about what they want from computers and what they do not want. They must have a clear idea of the changes which are acceptable and those which are not. They must devise means of promoting the desirable changes and minimizing the effects of the undesirable.

There should be a controlled and sequential staff development program conducted over a considerable period of time, eg. one year. During this time only limited classroom use should occur as teachers find their feet using the experiences of their mentors as starting points. Effort should be taken to ensure that students' first encounters with computers do not occur during mathematics lessons. Traditional areaa such as art and music offer much richer opportunities at least initially. Contemporary studies of business or commerce with the electronic office, manufacturing and workshop practice with robotics and numerical control, creative writing with word processors, together with computer awareness as a unit in social studies provide non-traditional contexts in which students can become familiar with computers. Studies of these devices for their own sakes should be undertaken in lessons or courses specifically allocated for that purpose. Many teachers have discovered the hard way that mathematics cannot provide the time, opportunities, funding or personnel to meet the broad range of services which computers spawn.

It is especially important that schools intending to purchase computers, decide how they will resolve the almost inevitable conflict between classroom (educational) use and administrative (office) use. Despite initial goodwill many administrations have become dominant. The development of a computing faculty and/or a computer services department is equally inevitable and the provision of personnel must be planned. Present laissez-faire policies have contributed to the growing shortages of teachers for mathematics classes.

Decisions must be made on educational grounds, a point which computer enthusiasts, salespeople and those with industrial and engineering backgrounds often ignore.

When the discussion deals with curriculum and learning matters it should be borne in mind that the computer will be used as a tool to assist with the acquisition or consolidation of specific skills and knowledge or to assist in carrying out some investigation. Both modes of use impose, on teachers and students, an obligation to be as certain as possible that the task set is being performed. In the long run improvements in the uptake of basic skills will no doubt attract the attention of academic researchers whilst for the time being it may continue to be assumed as an article of faith. When used as an investigative tool in school mathematics the student can attempt to verify the reliability of results by using methods such as estimation, pen and paper calculation, construction, simulation by other means and so on. No matter what the situation the user is responsible for the output. As will be expounded upon in Sub Chapter 6.1.5.

Much of the commercial software available for mathematics is of the test-tutor or drill-and-practice types. Whilst some of it does promote legitimate learning on the part of the student much of it does not. In some instances the user is trained to answer only certain questions, which appear every time the program is run, rather than to learn a more general skill or even form a concept. Others offer a little more in the form of nearly all the possible permutations in number fact recall. But often, looking at software with a teacher's eye, the impression given is that the exercise of the programmer's art was of much greater significance than fulfilling the learning needs of the user.

Fortunately it now seems that producers have recognized a buyer's market and that they are making the effort to produce software which challenges mathematics students to solve problems rather than merely providing answers. To their credit producers have taken teachers and curriculum writers into their teams. The marketplace no longer contains only what the programmer perceived as being mathematics. Exciting examples of software now support existing curricula in encouraging a wide range of skill and concept development and, more importantly, in serving as tools for investigation of problems rather than as suppliers of answers. Some of the materials worthy of consideration when beginning include ITMA from U.K. RIME from Victoria, Australia and that which accompanies the student text 'Maths for the real world' (ISBN 0 85896 0152). In each case learning is encouraged through illustration, demonstration, representation or calculation but always decisions are left with the learner. Furthermore, each is a component of a learning package which recognizes that students and teachers are also involved. There is little point in buying, or even being given, software which does not readily serve as a resource for the current curriculum unless the teacher has sufficient flexibility to accommodate it and, of course, it will meet the current needs of students.

Copyright difficulties, misunderstandings and infringments will persist. Legally and morally schools and teachers have an obligation to buy from legitimate sources. However licensing procedures must be developed to allow for limited copying of disks and tapes for use within an institution. Cartridge software should also be written to allow network use within the purchasing institution. Software to assist in investigations is often of greater value if it can be modified by the student, a practice forbidden under some legislation.

6.1.3.3 **Programming**

At the time of writing, programming is possibly the most contentious area in the use of computers in mathematics. It has been rightly argued that programming is a problem solving activity. In mathematics it is often as important to find a solution to a problem as it is to find one which is elegant or general. For schools the arguement about the form of programming in mathematics should be largely irrelevant. We do not discourage students from talking or writing because they are inarticualte or ungrammatical. In fact we urge them to practice their communication skills, believing that any use is better than none, and we attempt to help them to improve. A similar attitude should be taken in mathematics.

The purpose pf programming in mathematics is to enable students to show that they understand concepts and can apply them or to assist in an investigation. Whilst a well designed procedure may show greater understanding or involve less effort it should not be an over-riding consideration. What matters most is that the student is in control no matter how clumsily. When the time is appropriate the quality and level of control can be refined. If the use of a computer program shows a student a pattern which suggests the underlying structure of a solution then surely the computer has been as asset. If this pattern now enables the student to generate a solution independently of the computer then learning has been encouraged and extended. We must not discard the thrill of discovery.

6.1.4. Organizational aspects

Carol Cayley

6.1.4.1 Introduction

Because these aspects will depend greatly on the pertaining educational system and on the available equipment, a case study is included here to offer some points of consideration that may prove applicable or perhaps give rise to variations of organization.

The experiences of the case study are those of the Flagstaff Hill Primary School in Australia.

The school follows a system in which pupils complete a roster of weekly assignments in a self chosen order and at their own pace.

The teacher, Mrs Carol Cayley, relates the following.

One question I am often asked by teachers planning to use a computer in their classroom is how to make best use of the limited equipment they have available for the benefit of all students without having a nervous breakdown in the process. It is true that classroom organization becomes more difficult when you are trying to include computer oriented activities into the teaching program, but planning and forethought can avoid many of the pitfalls of such a venture. It is not only the teacher's routine that can be disturbed by the computer's arrival in the classroom but the student's routine as well.

There are a number of points to consider when planning this type of project. Some of them may seem very obvious to the reader, but from experience I can tell you that very often it is the obvious that we ignore as we concentrate on more professional education pursuits such as curriculum. However many of these can have a devastating effect on the success of failure of the programme.

6.1.4.2. Location

Locating the computer in your classroom is not always an easy task. You need to be able to see it to offer assistance where necessary, yet children need some privacy to learn from their mistakes. This is particularly important in the early stages if children are to be confident users of the computer.

Other children in the class should not be distracted from their tasks, but there may be times when a whole class demonstration is necessary. You may find a mobile trolley useful for this purpose. It is easy to turn the computer to face the class when necessary, but need not remain there indefinitely.

Window glare can make it difficult to read the screen. This problem may be increased at certain times of the day.

Chalk dust and discs are not compatible, so the computer should be housed in a chalk free area. Static electricity can also prove a problem, particularly if nylon carpet covers the floor of the classroom.

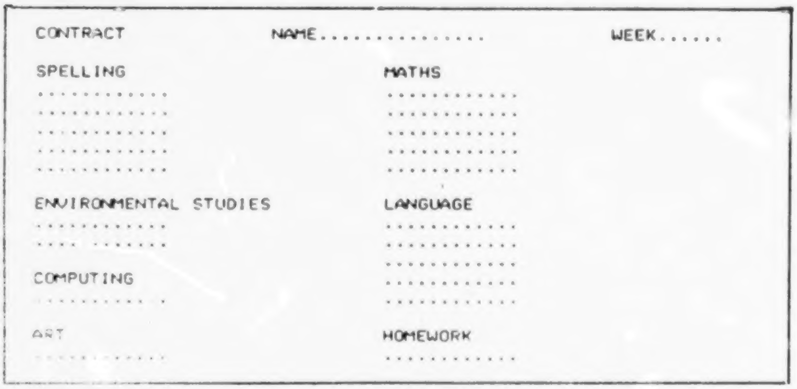
6.1.4.3 Scheduling

Finding time to fit computer use into a crowded schedule can be difficult. I found the following solution.

Students have a weekly "contract" of tasks to be completed in a self choosen order. This system allows students to develop some responsibility for their own actions and provides information for the teacher. The contract implies all kinds of work and only one in executed on the computer.

I have used this system successfully. All children were expected to complete a certain minimum amount of work. The format of the contract allowed for individual differences, and included computer activities as a separate heading. This was done, not to designate the computer as a separate and special area, but to ensure that all children had the opportunity for computer use, and to give the teacher some control over how that time was spent.

A simple form was designed to record this type of information.



CONTRACT NAME............... WEEK......

SPELLING
....
....
....
....
....

MATHS
....
....
....
....
....

ENVIRONMENTAL STUDIES
....
....

COMPUTING
....

ART
....

LANGUAGE
....
....
....
....
....

HOMEWORK
....

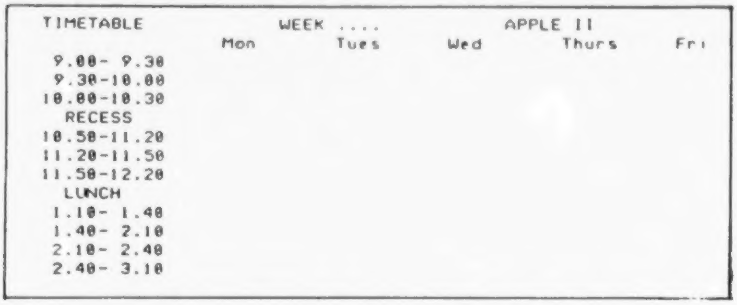
Figure 1

Within this system children can be directed to an activity by the teacher, and yet there is room for flexibility to develop their own ideas. Thus the teacher can set individual children specific tasks, develop remedial/extension/consolidation and still maintain some control over what happens in the classroom.

In the primary school there is not the degree of subject specilization so that each shild could have access to the computer for individual use for about 45 minutes per week, although soem children may get more, depending on the type of activities being used.

6.1.4.4 Timetable

In order to work out who should have access to the computer at what time, a simple method was developed using a large weekly timetable. Times not available due to set commitments, specialist teachers etc were marked out, and the available time divided into 30 minute slots.



TIMETABLE		WEEK		APPLE II	
	Mon	Tues	Wed	Thurs	Fri
9.00- 9.30					
9.30-10.00					
10.00-10.30					
RECESS					
10.50-11.20					
11.20-11.50					
11.50-12.20					
LUNCH					
1.10- 1.40					
1.40- 2.10					
2.10- 2.40					
2.40- 3.10					

Figure 2

Children were allocated a time by the teacher with some consideration given to the particular strengths and weaknesses of each child.

Children accepted the fact that using the computer was not an excuse for non-completion of a task (Contract provided a check). While one child was working at the computer, another child kept track of assignments, notices etc and recorded any necessary information.

A clock by the computer is useful, if all students don't have watches as it saves the teacher from continually having to check on the time. Alternatively a student can be appointed to keep time for others.

6.1.4.5. Aids for children

There are a variety of aids for children who find they need help. This stops them wearing a track to the teacher's desk, and frees the teacher from continual interruptions.

BEST COPY AVAILABLE

Children can make and contribute to a class reference book, using an alphabetical index for easy reference. Information can be included about anything the children feel is important e.g. LOADING a cassette. This gives students a reference point that is hopefully less complex than the computer's manual, but still of real value. Large charts can contain reference information as well.

Several students who are able to use the computer and their own initiative and can effectively communicate with their classmates can be designated "troubleshooters" to provide assistance initially. The teacher is then called as a last resort.

KEEPING TRACK

Written feedback is useful if the teacher wants to review student progress at a later date. Children could have a "Computer Book" for recording everything they do using the computer. This is not to designate the computer as a "special activity", but rather to simplify the teacher's task.

Specially designed activity cards can ensure the teacher receives the type of feedback needed with a minimum of disruption. This is particularly important when the software being used doesn't keep records for teacher use - or you don't have a disc drive.

6.1.4.6 Tasks

Activity cards can also be used to extend available software by providing tasks that do not necessarily require a computer. This is also a way of utilizing the motivational aspect of the computer. For example, if students enjoy playing the arcade style games available, they could then record their scores, graph them in various formats, and then interpret their graph. A dice game, such as Yahtzee, can be used with probability exercises.

Logic games on the microcomputer can provide a variety of skill levels, offering challenge without frustration. The use of games often has additional benefits in the spontaneous discussion, verbalization, peer teaching, group co-operation and problem solving that results from their use.

Simulation games can be used to develop concepts in a "real" situation, and extended within a "learning centre" to encompass other curriculum areas.

6.1.5. Software to Serve Students

Hartley Hyde

Too many schools choose a particular type of machine and then look for interesting software. We should first examine the school's curriculum needs, find the cariety of software required and then buy an appropriate machine. Problems relating to software choice are a vital consideration. This paper addresses one of these problems. We will examine the advantages of standardising educational software, but also recognise a need to diversify each child's learning experience.

To Serve Humans (Knight, 1974), is a short science fiction story, in which Adrian Knight described the appearance of benevolent aliens who provide humanity with many labour saving devices, with scant regard for humanity's needs. The high technology provided was identical for all nations, and all were pleased because it greatly surpassed that which they had built themselves.

The new technologies are a product of space research, and as far as many teachers are concerned, might just as well have been designed by aliens. The software to run these machines is so much better than the home-grown variety that there are great pressures to buy the best commercial product avaiable and conform to the trend being set by all those other teachers making the same decision.

6.1.5.1 Advantages of software standardisation

Standardisation of software is perceived desirable by the computing community in much the same way that mathematicians prefer standard notations. Attempts are constantly being made to standardise the features of computer languages. LOGO and Pascal are reasonably clearly defined while the varieties of BASIC are beyond control.

Some aspects of computer software cause frustration because of a lack of conformity - instance the variety of text editors and cursor controls available for use with each machine. One of the most infuriating examples is Apple LOGO. A strength of the BBC microcomputer is conformity os editing facilities, but adherents seem to get all the more annoyed if the conventions are broken.

There are many advantages if we all use the same software. I am more likely to find your LOGO notes of help in my classroom if the LOGOs we use are similar. Public domain programs which teachers and students develop for a system are more easily exchanged if common hardware and software systems are used.

Ballard (1983) has drawn our attention to studies which show that students in low socio economic enviroments are more likely to use drill and practice type software, while students from richer enviroments are more likely to be encouraged to write their own programs. There may be egalitarian pressures to avoid widening cultural and social gulfs by ensuring that all students receive a common computing experience.

Economic and copyright pressures are growing, and it may prove expedient for education systems to bulk buy hardware and arrange suitable software licensing agreements: all schools within a system sould then have exactly the same limited supply of resources. Roblyer (1983) reminds us that the costs of teacher developed software are also "unnecessarily high if teachers are unwilling to consoder using already-available software which is only slighlt different".

6.1.5.2 The need for diversity

Ted Kahn, director of education at Picodyne Corp., California, has asked:

"When are we going to stop looking for a single educational panacea that works for all kinds in all settings for all purposes? The challenge is to find what's best for each child in each learning enviroment."(Mace, 1984:49).

These remarks were made in criticism of a recent "strong push" to train all teachers to use LOGO - it is therefore all the more interesting to note an earlier statement attributed to Papert:

> Just as we must respect the individuality of a culture, we must respect the individuality of a child. Papert made a strong case for allowing children to gain knowledge in a way that suits their style of knowledge acquisition. He pointed out that different personality types tend to prefer different methods of learning. To prescribe one style of learning for all children is inefficient and harmful. Using mathematics as an example, Papert pointed out that those considered good in math are those who learn easily from whatever style is currently used. (Coon, 1982:14)

6.1.5.3 Differences among students

Individual differences among students are traditionally described using continuums such as intelligence, attitudes measures, anxiety levels, flexibility, achievement motivation, learning speed and various socio-economic factors.

Useful studies which relate Computer-Assisted Instruction learning rates to individual differences were reviewed by Reid _et al_. (1973). It is time to pursue similar studies using more recent software.

Many studies have considered questions of suitability of various types of software relating to sex differences. Researchers measure correlations between criterion measures and student's sex differences which may explain their observations - it may be more helpful to distinguish appropriate underlying continuums: not all boys are aggressive in their search for knowledge, and not all girls prefer to work in co-operative groups.

6.1.5.4 Different types of software

Schultz and Hart (1983) prepared a chart which clearly described a classification of educational software for which they first of all distinguished between the study of computers, and a study using computers. When reviewing Australian software, Anderson (1984) chose to use the classification tutor/tool/tutee attributed to Taylor (1980). Margitta (1979) distinguished between an author directed machine, a learner directed machine and a learner controlled machine.

Most Tutorial or Drill and Practice programs can be described as Author directed. The Educational Software Evaluation Consortium (1983) listed 156 mathematics titles receiving favourable reviews, of which 123 titles (79 per cent) were described as either drill and practice or tutorial.

Learner directed programs include Microworlds and Building Sets. An excellent example is Rocky's Boots (Robinett, 1982; Piestrup, 1984) in which children are able to build and manipulate simple logic circuits. MuMATH-83 (Stoutemyer, 1983) is a much more complicated example which many would classify as a computing language.

Greater learner control is achieved when a relatively large amount of time is invested in teaching children to program. Popular languages for mathematics classes include LOGO, FORTH, APL, AMPL, Pascal, and possibly BASIC. Unless we are careful, this investment can prove counterproductive. Many LOGO books recommend the construction of equilateral triangles as an early exercise. Students learn that they need to make three right turns, each of 120 degrees - this may help them to learn to program, but may not teach good mathematics (Leron, 1984). With the teaching skills each of us have developed over the years, we can often teach concepts more effectively, and in much less time, if we leave the computer swithced off.

6.1.5.5 Matching software and students

Suppes (1966) suggested that:

> ...in a few more years millions of school-children will have access to what Phillip of Macedon's son Alexander enjoyed as a royal perogative: the personal services or a tutor as well-informed and responsive as Aristotle.

Today's software may be well informed and responsive, but falls far short of the skills of a personal tutor. We have neither sufficient software to present a total curriculum, nor the variety required t o cater for individual differences. So long as machines become superseded so quickly, we are unlikely to have sufficient software to match our equipment. We may make more software progress if we were content to use outdated hardware.

Even if we eventually have sufficient software, tomorrow's challenge is to select software most suitable for individual students at each point of their progress through the curriculum. It is not difficult to imagine an intelligent management system whcih could maintain sufficient records of student's progress and individual differences to select appropriate software. We already have the technology, but we don't have the software or the research base with which to program the artificial intelligence.

6.1.5.6 A recipe book approach

The science fiction story cited above concludes by describing the imminent departure of humans on a sightseeing tour of the alien's planet. With appropriate timing, the hero realises that an alien text book entitled How To Serve Humans, is really a recipe book.

The pun on the word "serve" is just as appropriate to the present problem as it was to the original story. There is a real sense in which we prepare students for consumption by society, and we should be cognisant of the needs of society as well as the needs of the individual. What service are we providing future industry and graphic design, if all students approach computer graphics with a turtle mentality? How much do we limit the imagination of future mathematicians if their collective software experience is narrow?

Whenever the community of mathematics teachers meets a new challenge, or prepares itself to teach new material, there is a strong lobby to adopt a recipe book approach to teaching. When an enthusiast discovers a methodology that works, other teachers are only too ready to copy the methodology step by step.

6.1.5.7. Learner controlled software

When Apple Logo was first released, Harold Ableson's (1982) book which provided a teaching sequence, proved more popular than the reference manuals which came with the software (Davidson, 1982). For some time teachers followed the Ableson recipe and considerable conformity was evident. Since 1982, the number of methodology books for LOGO has increased dramatically. We can easily discover a wide variety of teaching techniques using LOGO: the recipe book approach now encourages diversity instead of conformity.

Experimentation and growing confidence will eventually allow teachers to develop their own techniques. Clements (1983) described a teaching technique in which students were encouraged to write their own drill and practice programs using LOGO procedures. In this way students were able to see the variety of uses for the software, and gain considerable enjoyment and reward form what might otherwise have proved a boring drill routine.

MuMATH appears to have some potential to modify our approach to algebra in much the same way that calculators have influenced our approach to arithmetic. Where are the magazine articles and books suggesting ways of using muMATH? While a well researched book on the use of muMATH would have a very powerful influence on the teaching of algebra, the initial from mathematics teachers would precipitate an avalanche of similar, and probaly improved, products - conformity may prove an advantage if large groups of teachers identify the same needs.

6.1.5.8 Author directed software

In magazine articles and papers such as this, it is fashionable to downgrade the importance of simple conditioning programs: other types of software are much more exciting to describe. Because home-grown software doesn't have to be written for the wide variety of systems available, it will often fail if used on slightly different equipment in another school. Because teachers can always explain how to use their own software there is less need for detailed documentation. Teacher developed software has earned a poor reputation, and it is commercially expedient to highlight the disadvantages - teachers who write their programs to suit their own teaching style, don't pay bills for software houses.

We can use our own software to teach students essential, tedious drill and practice lessons in an interesting and memorable way at a fraction of the cost of commercial products. Robert Gagne (Lipsitz, 1982) claims that:

...basic procedures of reading and arithmetic - ought to be not simply mastered, but brought to a state of automaticity.

...If the teacher finds drill and practice to be a time-sonsuming and dreary activity, so far as she is concerned, then that is the place to use the computer.

6.1.5.9 Summary

This paper has considered some advantages and disadvantages of standardising educational software. If such a discussion is to prove valuable, it must be considered at the time of first setting up a school computing system. Rory Butler has drawn our attention this week, to an analogy between choosing a system and digging a ditch. The more we invest our time and money, the deeper we dig the ditch. Whether we choose to dig separate ditches, or one deeper ditch for everybody, that must be an early decision.

Central to the whole question are the diverse needs of individual students, and the social-evolutionary advantages of maintaining as much diversity in the population as possible. Balanced against this are the economic and time constraints involved in each doing our own thing completely alone. We need enough people in each ditch to produce the variety of software needed. We need to think very carefully before we allow commercial interests to discover those needs for us because we may find that some commercial interests are alien to our purpose.

6.1.5.(10). References

H Abelson Apple Logo. Peterborough: BYTE/McGraw-Hill, 1982.
J Anderson Computing in Schools. Hawthorn: The Australian Council for Educational Research, 1984.

A Ballard Computer in Early Childhood Education. Early Childhood Education (3-8 Years) Newsletter. 2, 12-13, 1983.

D H Clements Programming, Problem Solving, and Practice. Arithmetic Teacher. 31 (4), 32-35, 1983.

M Coon Papert at the Faire. The Computing Teaher. 11, 14-16, 1984.

L J Davidson Apple Log. Quebec: Logo Computer Systems, 1982. Educational Software Evaluation Consortium. The 1983 educational sofware preview guide: mathematics. The Journal of Computers in Mathematics and Science Teaching. 3, 34-38, 1983.

D Knight To Serve Man. In J Haldeman. Cosmic Laughter; Science Fiction for the Fun of it. New York: Holt, Rinehart and Winston, 1974.

U Leron Some problems in children's LOGO learning. Unpublished paper, 1984.

L Lipsitz Developments in Learning Psychology - An Interview with Robert Gagne. Educational Technology. 22, 11-15, 1980.

S M Mace Where is Logo Taking Our Kids? InfoWorld. 6 (4), 46-50, 1984.

I Margitta Learning with Computers. Curriculum Research Bulletin. 14, 2-13, 1979.

A Piestrup Game Sets and Builders. BYTE. 9 (6), 215-219, 1984

J B Reid, R L Palmer, J Whitlock & J Jones Computer-Assisted Instruction Performance of Student Pairs as Related to INdividual Differences. Journal of Educational Psychology. 65 (1), 65-73, 1973.

W Robinett Rocky's Boots. Menlo Park: The Learning Company, 1982.

M D Roblyer The Case for the Against Teacher-Developed Microcomputer Courseware. Educational Technology. 23 (1), 14-17, 1983.

K A Shultz & L C Hart Computers in Education. Arithmetic Teacher. 31 (4), 36-37, 1983.

D Stoutemyer The muMATH-83 Symbolic Mathematics Sysytem Reference Manual. Honolulu: The Soft Warehouse, 1983.

P Suppes The Uses of Computers in Education. Scientific American. 215, 206-221, 1966.

R P Taylor (Ed.) The Computer in the School: Tutor, Tool, Tutee. New York: Teachers College Press, 1980.

6.2 Technology for Education of the Handicapped. This group did not attract participants and was therefore cancelled.

6.3 Technology as an Aid to the Teaching of Problem Solving.

(reported under the ICME publication Problem Solving)